HISTORY OF STEEL BEAM-COLUMN EQUATIONS

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INTRODUCTION

Design guidance for structural steel members under combined axial and transverse loads, or beam-columns, has evolved from very simple assumptions to the present ultimate-strength approaches. The changes in design recommendations have paralleled the state of knowledge of the behavior of these structural elements. Early elastic models have given way to an understanding of inelastic behavior. This paper follows the advances in design recommendations for these members from the late 19th century to the present specifications, including load and resistance factor design. This information on past design methods will be of assistance to structural engineers involved with rehabilitation of vintage structures.

EARLY DESIGN METHODS

Earlier generations of engineers understood the need to somehow combine the effects of axial load causing direct stress and lateral load causing bending stress. The simplest approach is to use the standard strength of materials elastic solution for combined stress of:

$$f_{combined} = f_{axial} + f_{bending} \qquad (1)$$

It was recognized quite early by most engineers that the resulting deflections caused by the lateral loads tended to create additional moment because of moment magnification by what is presently referred to as the "P-delta" effect. Many authors (Burt 1917; Fuller and Kerekes 1931; Johnson et al. 1897; Merriman and Jacoby 1911; Shedd 1934; Spofford 1915) and some early specifications (Waddell 1912) made an attempt to provide for this magnified moment in their recommended interaction equations. The usual approach was an attempt to take into account the moment magnification caused by the lateral displacement of the beam-column by the transverse loads. Ketchum's (1921) approach, as listed in the following, was the interaction method in general use by many engineers during the era before about 1935:

> 44. Combined Stress--Members subject to combined direct and bending stresses shall be proportioned to the following formula:
>
> $S = [P/A] + Mc/[I- (PL^2/10E)]$
>
> where:
>
> | | | |
> |---|---|---|
> | S | = | stress in lb. per sq. in. in extreme fiber; |
> | P | = | direct load in pounds; |
> | A | = | area of member in sq. in.; |
> | M | = | bending moment in in. lb.; |
> | c | = | distance from neutral axis to extreme fiber in inches; |
> | I | = | moment of inertia of member; |
> | L | = | length of member, or distance from point of zero moment to end of member in inches; |
> | E | = | modulus of elasticity = 30,000,000 lb. per sq. in. (Ketchum 1921) |

The derivation of this equation, along with modifications recommended by other authors, is shown in Appendix I.

Some authors advocated an increase in allowable stress for cases of combined stresses, over and above the usual 33% (or as typical of this period, 50%) allowable stress increase for loads combined with wind. Burt (1917) "… allow(s) three-fourths of the bending moment to be used..." while Ketchum (1921) states, "When combined stress … is considered, 25 percent may be added to the allowable stresses." The origin of these provisions is unknown and apparently was not widespread as no other cases where authors recommended this could be found.

AISC ELASTIC PROCEDURES

Early AISC specifications *(Specification* 1928) gave no required interaction relationship, but instead stated the requirement in general terms.

> Section 10. Combined Stresses.
> Members subject to both direct and bending stresses shall be so proportioned that the greatest combined stresses shall not exceed the allowable limits." *(Specification* 1928)

The designer was presumably free to choose his preferred form of interaction equation. Some authors of this era (Fuller and Kerekes 1931; Shedd 1934) still recommended the use of the interaction equations listed in Appendix I.

With the 1936 revision of the AISC specification *(Specification* 1936), the straight-line interaction equation was officially adopted for use in buildings.

> Section 6. Combined Stresses.
> (a) Axial and Bending.
> Members subject to both axial and bending stresses shall be so proportioned that the quantity:
>
> $f_a / F_a + f_b / F_b$
>
> shall not exceed unity, in which:
>
> F_a = axial unit stress that would be permitted by this Specification if axial stress only existed.
>
> F_b = bending unit stress that would be permitted by this Specification if bending stress only existed.
>
> f_a = axial unit stress (actual) = axial stress (sic.) divided by area of member.

f_b = bending unit stress (actual) = bending moment divided by section modulus of member. *(Specification* 1936)

Although the straight-line interaction equation did not consider secondary moments, it did perform satisfactorily for many years because the working stresses of this era were low by current standards. Grade A-7 steel had a yield strength of 33 ksi. The basic compression working stress was 17 ksi or 0.51 F_y and the basic bending working stress was 20 ksi or 0.60 F_y. By keeping the compressive stress low, the secondary moments were also kept manageable.

With the 1961 revision of the AISC specification *(Specification* 1961), the straight-line interaction equation for combined axial and bending stress was replaced by a pair of equations that were the result of extensive work in the area of ultimate strength of beam-columns. These equations are commonly referred to within the profession as the stability equation and the strength equation. They are presented here, simplified for bending in only one plane:

$$\frac{f_a}{F_a} + \frac{C_m f_b}{(1 - f_a/F'_e)F_b} \leq 1.0 \qquad (2)$$

$$\frac{f_a}{0.60F_y} + \frac{f_b}{F_b} \leq 1.0 \qquad (3)$$

Eq. 2 (known as the stability equation) considered the stability of the beam-column and considers the effects of secondary moments developed as a result of P-delta effects. Eq. 3 (known as the strength equation) considered the strength of the beam-column at any point along its length. It should be noted that the value of F_b in (3) must consider the effects of possible lateral-torsional buckling of the beam-column as a beam. The use of the straight-line interaction equation is still allowed when $f_a/F_a \leq 0.15$, since the effect of secondary moment is small when the axial load is low.

Eqs. (2) and (3) have provided satisfactory service over the years and were retained into the last AISC allowable stress design specification *(Specification* 1990).

For the case of combined axial tension and bending, (4) has been listed in the AISC-ASD specification since 1961, with the stipulation that the computed bending stress arising from an independent load source relative to axial tension shall not exceed that allowed by bending alone *(Specification* 1961)

$$\frac{f_T}{F_T} + \frac{f_b}{F_b} \leq 1.0 \qquad (4)$$

ULTIMATE STRENGTH DESIGN

After World War II, research into the ultimate strength of steel structures began in earnest in the United States. Much of this work, which was centered at Lehigh University, Bethlehem, Pa., was funded by the Column Research Council and focused on the capacity of columns under axial thrust and flexure, or beam-columns.

The research produced two general equations for the ultimate strength of beam-columns. They have been stated in different forms by different researchers and are shown here in one basic form. *(Plastic* 1971)

$$\frac{P}{P_{cr}} + \frac{M_{eq}}{M_p(1 - P/P_e)} \leq 1.0 \qquad (5)$$

$$\frac{P}{P_y} + \frac{0.85M_2}{M_p} \leq 1.0 \qquad (6)$$

where:

P = applied load;

P_{cr} = critical column load in the absence of bending, but including influence of effective length, residual stress, and yielding;

M_{eq} = equivalent bending moment;

M_p = section plastic moment capacity;

P_e = column elastic buckling load (Euler column load);

P_y = column yield load;

M_2 = largest end moment.

Eq. (5) is derived from a laterally unsupported member and therefore considers stability of the beam-column (Johnson 1971). Eq. (6) considers the formation of a plastic hinge at the end of a member, and as such, the member is considered adequately braced to develop the plastic moment (Beedle 1964).

AISC initially adopted (5) and (6) in a set of draft rules for plastic design *(Plastic* 1959). The plastic design format is a load-factor design method, using weighted loads compared to the ultimate strength of the structure. This format was modified slightly for official adoption in the 1961 Specification *(Specification* 1961) as follows:

> Except as otherwise provided in this section, *Mo/Mp,* the ratio of allowable end moment to the full plastic bending strength of columns and other axial loaded members, shall not exceed the value given by the following formulas, where they are applicable:
>
> CASE I. For columns bent in double curvature by moments producing plastic hinges at both ends of the columns
>
> $M_o = M_p$ when $P/P_y \leq 0.15$
>
> $M_o/M_p \leq 1.18 - 1.18(P/P_y) \leq 1.0$ when $P/P_y > 0.15$
>
> CASE II. For pinned base columns required to develop a hinge at one end only, and double curvature columns required to develop a hinge at one end when the moment at the other end is less than the hinge value
>
> $M_o/M_p \leq B - G(P/P_y) \geq 1.0$
>
> the numerical values for B and G, for any given slenderness ratios in the plane of bending *l/r,* being those listed in Tables 4-33 and 4-36 of the Appendix. Where *l/r*

in the plane of bending is less than 60, and P/P_y does not exceed 0.15, the full plastic strength of the member may be used ($M_o = M_p$)

CASE III. For columns bent in single curvature

$M_o/M_p \leq 1.0 - H(P/P_y) - J(P/P_y)^2$

the numerical values for H and J being those given in Tables 5-33 and 5-36 of the Appendix. *(Specification* 1961)

This initial adoption was quite complicated and limited to steel with yield strengths of 33 ksi or 36 ksi. AISC simplified the format in 1970 *(Specification* 1970), changing the load factors and allowing higher strength steels. These equations in effect through the 1990 *Specification*.

The 1990 *Specification* (1990) lists the two plastic design equations as

$$\frac{P}{P_{cr}} + \frac{C_m}{(1 - P/P_e)M_m} \leq 1.0 \qquad (7)$$

$$\frac{P}{P_y} + \frac{M}{1.18M_p} \leq 1.0 \qquad (8)$$

where:

C_m = sway and end moment coefficient;

M_m = maximum moment that may be resisted in absence of axial load.

CURRENT LOAD AND RESISTANCE FACTOR DESIGN (LRFD) AND ALLOWABLE STRENGHT (ASD) EQUATIONS

The 1986 Load and Resistance Factor Design (LRFD) Specification adopted the current state-of-the art design philosophy for the design of steel structures. These design interaction equations are written so that the moment magnification terms can be removed if the designer prefers to perform an exact second-order structural analysis. The advantage of this will become more apparent in the future as second-order analysis methods become more widespread.

The development of the equations allowed the use of only one equation to be used, rather than the previous strength and stability checks. The interaction equations are as follows, from the 2010 *Specification*:

For $P_r/P_c \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_r}{M_c}\right) \leq 1.0 \qquad (9)$$

For $P_r/P_c < 0.2$

$$\frac{P_r}{2P_c} + \left(\frac{M_r}{M_c}\right) \leq 1.0 \qquad (10)$$

Since the thrust of the present paper is on history, the reader is referred to any of several current excellent references for further information on this current design method (Salmon and Johnson 1990; Chen and Lui 1985).

CONCLUSION

The present paper has reviewed the history of design of steel beam columns in American practice. This history has paralleled the state of knowledge regarding steel structures. It is hoped that the information contained herein would be useful to structural engineers involved in the structural retrofit of vintage buildings. A knowledge of past design practices can assist in understanding existing structural systems.

APPENDIX I. DERIVATION OF ELASTIC INTERACTION EQUATION

The elastic interaction equation

$$S = \frac{P}{A} + \frac{Mc}{I \pm \frac{kPL^2}{E}} \tag{11}$$

was in common use until the early 1930s. This equation considers the additional moment caused by the lateral deflection of the member under the influence of some transverse load. The derivation of (11) is found in many references, but the derivation presented here is similar to that found in Shedd (1934).

The effective second-order moment M' is the combination of the primary moment M plus the second-order effect of lateral displacement, as shown in Fig. 1.

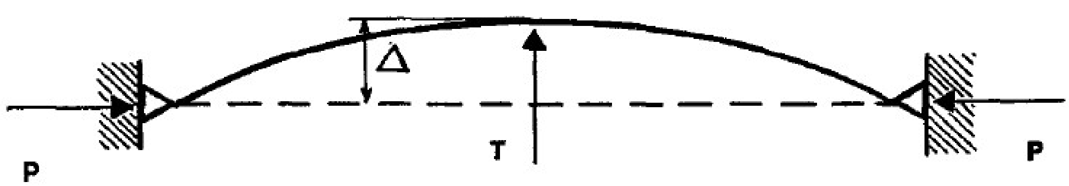

FIG. 1. Elastic Beam-Column Deflected Shape

$$M' = M + P\Delta \tag{12}$$

Considering the member as a beam, the deflection of the member can be stated as

$$\Delta = \frac{kM'L^2}{EI} \tag{13}$$

Combining (12) and (13) produces

$$M' = M + \frac{PkM'L^2}{EI} \tag{14}$$

Multiplying (14) by *c/I* converts the moments to flexural stress

$$\frac{M'c}{I} = \frac{Mc}{I} + \frac{PkM'L^2c}{EI^2} \tag{15}$$

Rearranging (15) produces

$$S = \frac{Mc}{I - \frac{kPL^2}{E}} \tag{16}$$

To this must be added the effects of direct axial stress *P/A* to produce

$$S = \frac{P}{A} + \frac{Mc}{I - \frac{kPL^2}{E}} \tag{17}$$

When axial tension is present, (17) is modified as follows to take into account the beneficial effects of axial tension which reduce the lateral deflection.

$$S = \frac{P}{A} + \frac{Mc}{I + \frac{kPL^2}{E}} \qquad (17)$$

where k is a constant, derived from basic deflection equations, as shown in Table 1. For practical purposes, k = 1/10 for pinned-ended members and 1/32 for members with fixed ends (Shedd 1934).

Table 1. Deflection Factors

Deflection coefficient k (1)	End fixity (2)	Distribution of loads (3)
5/48	Pinned	Uniform load
1/12	Pinned	Concentrated load at center
23/216	Pinned	Equal concentrated loads at 1/3 points
19/192	Pinned	Equal concentrated loads at 1/4 points
1/16	Fixed	Uniform load
1/24	Fixed	Concentrated load at center
5/72	Fixed	Equal concentrated loads at 1/3 points
1/18	Fixed	Equal concentrated loads at 1/4 points

APPENDIX II. REFERENCES

Beedle, L. S. (1964). *Structural steel design.* Ronald Press, New York, N.Y.

Bjorhovde, R. (1988). "Columns: From theory to practice." *AISC Engrg., J.,* 25(1), 21-34.

Burt, H. J. (1917). *Steel construction.* American Technical Society, Chicago, Ill.

Chen, W. F., and Lui, E. M. (1985). "Columns with end restraint and bending in load and resistance factor design." *AISC Engrg. J.,* 22(3), 105-132. 555

Fuller, A. H., and Kerekes, F. (1931). *Analysis and design of steel structures.* Van Nostrand, New York, N.Y.

Galambos, T. V. (1977). "History of steel beam design." *AISC Engrg., J.,* 14(4), 141-147.

Johnson, B. G., ed. (1971). *Guide to design criteria for metal compression members.* 2nd Ed., John Wiley & Sons, New York, N.Y.

Johnson, J. B., Bryan, C. W., and Turneaure, F. E. (1897). *Theory and practice of modern framed structures.* John Wiley & Sons, New York, N.Y.

Ketchum, M. S. (1921). *The design of steel mill buildings.* McGraw-Hill, New York, N.Y.

Load and resistance factor design specification for structural steel buildings. (1986). American Institute of Steel Construction, Chicago, Ill.

Merriman, M., and Jacoby, H. S. (1911). *A textbook on roofs and bridges, part I.* John Wiley & Sons, New York, N,Y.

Plastic design in steel. (1959). American Institute of Steel Construction, New York, N.Y.

Plastic design in steel, a guide and commentary. (1971). ASCE, New York, N.Y.

Salmon, C. G., and Johnson, J. E. (1990). *Steel structures, design and behavior.* *3*rd Ed., Harper and Row, New York, N.Y.

Shedd, T. C. (!934). *Structural design in steel.* John Wiley & Sons, New York, N.Y.

Specification for the design, fabrication, and erection of structural steel for buildings. (1928). American Institute of Steel Construction, New York, N.Y.

Specification for the design, fabrication, and erection of structural steel for buildings. (1936). American Institute of Steel Construction, New York; N.Y.

Specification for the design, fabrication, and erection of structural steel for buildings. (1961). American Institute of Steel Construction, New York, N.Y.

Specification for the design, fabrication, and erection of structural steel for buildings. (1970). American Institute of Steel Construction, New York N.Y.

Specification for structural steel buildings. (1990). American Institute of Steel Construction, Chicago, Ill.

Specification for structural steel buildings. (2010). American Institute of Steel Construction, Chicago, Ill.

Spofford, C. M. (1915). *Theory of structures.* McGraw-Hill, New York, N.Y.

Waddell, J. A. L. (1912). *Specifications for steel bridges.* John Wiley & Sons, New York, N.Y.

<u>QUIZ</u>

HISTORY OF STEEL BEAM-COLUMN EQUATIONS
1 PDH

1. Moment amplification in beam-columns is caused by _________ .

    a. lateral displacement
    b. axial stresses
    c. flexural stresses
    d. weak axis buckling

2. The 1936 AISC Specification ________.

    a. considered second order moments
    b. did not consider second order moments
    c. provided a parabolic interaction equation
    d. used the Johnson parabola.

.3 The 1961 AISC Specification ________.

    a. considered second order moments
    b. did not consider second order moments
    c. provided a parabolic interaction equation
    d. used the Johnson parabola.

4. C_m is a ______________.

    a. sway and end moment coefficient
    b. direct buckling coefficient
    c. axial load modifier
    d. moment gradient modifier

5. Current AISC Specifications require the use of ________ equation for beam-column strength checks.

    a. 1
    b. 2
    c. 3
    d. 4