

Distribution of Stresses in Soil

By

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1. INTRODUCTION

1.1 SCOPE. This publication covers the analysis of stress conditions at a point, stresses beneath structures and embankments, and empirical methods for estimating loads on buried pipes, conduits, shafts, and tunnels.

1.2 STATE OF STRESS. Stresses in earth masses are analyzed using two basic and different assumptions. One assumes elastic conditions, and the other assumes full mobilization of shear strength (plastic equilibrium). Elastic solutions apply to problems for which shear failure is unlikely. If the safety factor against shear failure exceeds about 3, stresses are roughly equal to values computed from elastic theory. Plastic equilibrium applies in problems of foundation or slope stability and wall pressures where shear strength may be completely mobilized.

2. STRESS CONDITIONS AT A POINT

2.1 MOHR'S CIRCLE OF STRESS. If normal and shear stresses at one orientation on an element in an earth mass are known, stresses at all other orientations may be determined from Mohr's circle. Examples of stress transformation are given in Figure 1.

2.1.1 PLASTIC EQUILIBRIUM. The use of Mohr's circle for plastic equilibrium is illustrated by analysis of triaxial shear test results.

2.2 STRESSES IN SOILS. The normal stress at any orientation in a saturated soil mass equals the sum of two elements: (a) pore water pressure carried by fluid in soil spaces, and (b) effective stress carried by the grain skeleton of the soil.

2.2.1 TOTAL STRESS. The total stress at any point is produced by the overburden pressure plus any applied loads.

2.2.2 PORE WATER PRESSURE. Pore water pressure may consist of (a) hydrostatic pressure, (b) capillary pressure, (c) seepage or (d) pressure resulting from applied loads to soils which drain slowly.

2.2.3 EFFECTIVE STRESS. Effective stress equals the total stress minus the pore water pressure, or the total force in the soil grains divided by the gross cross-sectional area over which the force acts.

2.2.4 OVERBURDEN PRESSURE. Division of weight of overlying soil and water into effective stress and pore water pressure depends on the position of the groundwater table or the flow field induced by seepage. For static water condition, effective stresses at any point below the groundwater level may be computed using the total unit weight of soil above the water level and buoyant unit weight below the water level. Pore water pressure is equal to the static head times the unit weight of water. If there is steady seepage, pore pressure is equal to the piezometric head times the unit weight of water,

and the effective stress is obtained by subtracting the pore water pressure from the total stress.

2.2.5 APPLIED LOAD. Division of applied load between pore pressure and effective stress is a function of the boundary conditions, the stress-strain properties, and the permeability of the stressed and surrounding soils. When drainage of pore water is inhibited, load is compensated for by increased pore water pressures. These pressures may decrease with time, as pore water is drained and load is transferred to the soil skeleton, thereby increasing effective stress.

2.2.6 EFFECTS OF STRESSES ON A SOIL MASS. Analysis of a soil system (e.g., settlement, stability analyses) are performed either in terms of total stresses or effective stresses. The choice between the two analysis methods is governed by the properties of the surrounding soils, pore water behavior, and the method of loading.

3. STRESSES BENEATH STRUCTURES AND EMBANKMENTS

3.1 SEMI-INFINITE, ELASTIC FOUNDATIONS

3.1.1 ASSUMED CONDITIONS. The following solutions assume elasticity, continuity, static equilibrium, and completely flexible loads so that the pressures on the foundation surface are equal to the applied load intensity. For loads of infinite length or where the length is at least 5 times the width, the stress distribution can be considered plane strain, i.e., deformation occurs only in planes perpendicular to the long axis of the load. In this case stresses depend only on direction and intensity of load and the location of points being investigated and are not affected by elastic properties. Shearing stresses between an embankment and its foundation are neglected.

3.1.2 STRESS DISTRIBUTION FORMULAS. Figure 2 presents formulas based on the Boussinesq equations for subsurface stresses produced by surface loads on semi-infinite, elastic, isotropic, homogeneous foundations. Below a depth of three times the width of a square footing or the diameter of a circular footing, the stresses can be approximated by considering the footing to be a point load. A strip load may also be treated as a line load at depths greater than three times the width of the strip.

3.1.3 VERTICAL STRESSES BENEATH REGULAR LOADS. Charts for computations of vertical stress based on the equations are presented in Figures 3 through 7. Use of the influence charts is explained by examples in Figure 8. Computation procedures for common loading situations are as follows:

3.1.3.1 SQUARE AND STRIP FOUNDATIONS. Quick estimates may be obtained from the stress contours of Figure 3. For more accurate computations, use Figure 4.

3.1.3.2 RECTANGULAR MAT FOUNDATION. For points beneath the mat, divide the mat into four rectangles with their common corner above the point to be investigated. Obtain influence values I for the individual rectangles from Figure 4, and sum the values

to obtain the total I. For points outside the area covered by the mat, use superposition of rectangles and add or subtract appropriate I values to obtain the resultant I. (See example in Figure 9.)

3.1.3.3 UNIFORMLY LOADED CIRCULAR AREA. Use Figure 5

3.1.3.4 EMBANKMENT OF INFINITE LENGTH. Use Figure 6 for embankments of simple cross section. For fills of more complicated cross section, add or subtract portions of this basic embankment load. For a symmetrical triangular fill, set dimension b equal to zero and add the influence values for two right triangles.

3.1.3.5 SLOPING FILL OF FINITE DIMENSION. Use Figure 7 for stress beneath the corners of a finite sloping fill load.

3.1.3.4 VERTICAL STRESSES BENEATH IRREGULAR LOADS. Use Figure 10 for complex loads where other influence diagrams do not suffice. Proceed as follows:

(1) Draw a circle of convenient scale and the concentric circles shown within it. The scale for the circle may be selected so that when the foundation plan is drawn using a standard scale (say 1"=100'), it will lie within the outer circle.

(2) Plot the loaded area to scale on this target with the point to be investigated at the center.

(3) Estimate the proportion A of the annular area between adjacent radii which is covered by the load.

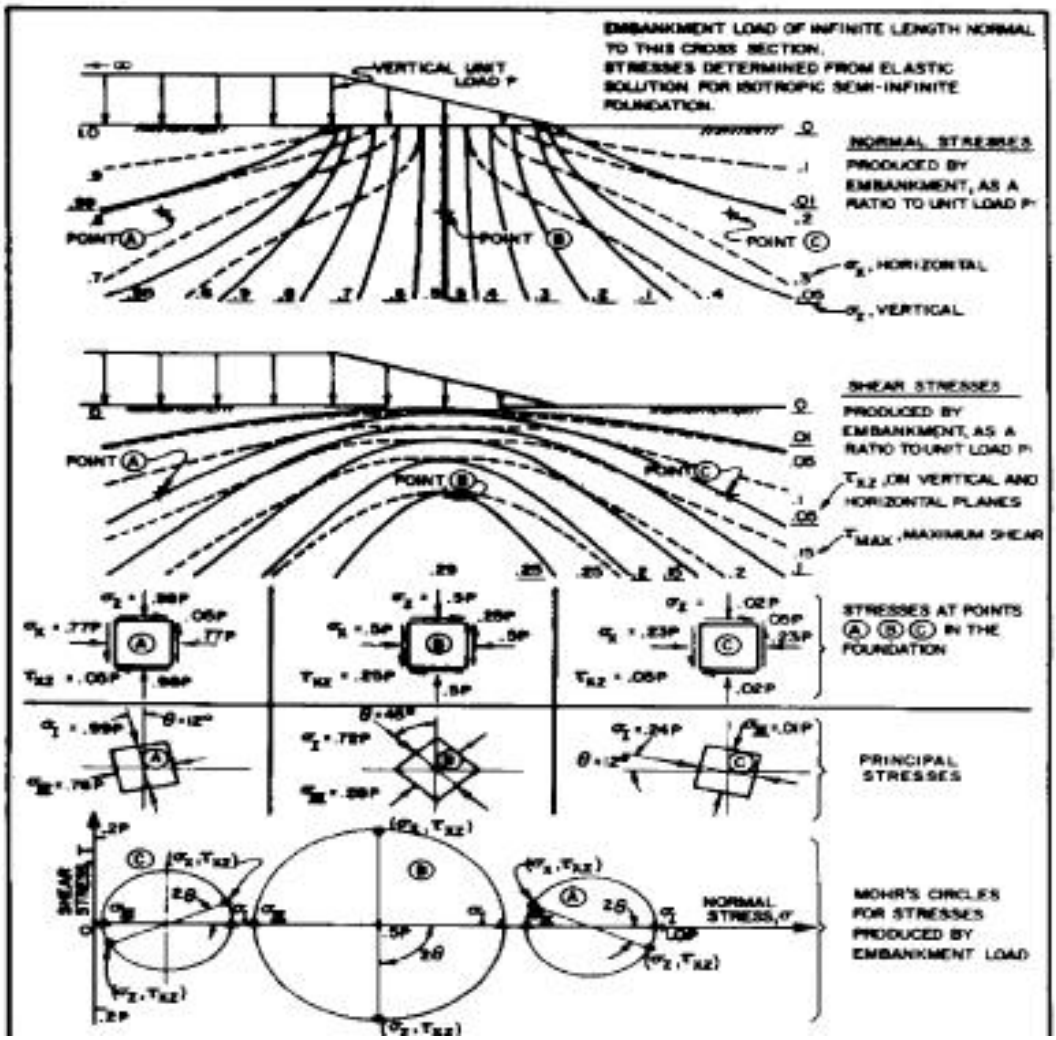


Figure 1

Examples of stress conditions at a point

LOADING CONDITION	STRESS DIAGRAM	STRESS COMPONENT	EQUATION
POINT LOAD		VERTICAL	$\sigma_z = -\frac{P}{2\pi R^2}\left[\frac{3z^3}{R^3} + \frac{(1-2\mu)R}{R+z}\right]$
		HORIZONTAL	$\sigma_r = \frac{P}{2\pi}\left[3\frac{r^2 z}{R^5} - (1-2\mu)\frac{R}{Rr^2}\right]$
		SHEAR	$\tau_{rz} = \frac{3P}{2\pi}\cdot\frac{rz^2}{R^5}$
UNIFORM LINE LOAD OF INFINITE LENGTH		VERTICAL	$\sigma_z = \frac{2p}{\pi}\cdot\frac{z^3}{R^4}$
		HORIZONTAL	$\sigma_x = \frac{2p}{\pi}\cdot\frac{rx^2}{R^4}$
		SHEAR	$\tau_{xz} = \frac{2p}{\pi}\cdot\frac{rz^2}{R^4}$
UNIFORMLY LOADED RECTANGULAR AREA (FIGURE 4)		VERTICAL (BENEATH CORNER OF RECTANGLE)	$\sigma_z = \frac{p}{4\pi}\left[\frac{2xyz\sqrt{x^2+y^2+z^2}}{z^2(x^2+y^2+z^2)} \cdot \frac{x^2+y^2+2z^2}{(x^2+z^2)(y^2+z^2)}\right.$ $\left. + \tan^{-1}\frac{2xyz\sqrt{x^2+y^2+z^2}}{z^2(x^2+y^2+z^2)-x^2y^2}\right]$
UNIFORMLY LOADED CIRCULAR AREA (FIGURE 5)		VERTICAL	$\sigma_z = p\left\{1 - \frac{1}{[1+(r/z)^2]^{3/2}}\right\}$
		HORIZONTAL	$\sigma_r = \frac{p}{2}\left[1+2\mu-2(1+\mu)\left(\frac{z}{\sqrt{r^2+z^2}}\right)+\left(\frac{z}{\sqrt{r^2+z^2}}\right)^3\right]$
		SHEAR	$\tau_{rz} = 0$ (STRESS COMPONENTS $\sigma_t, \theta, \tau_{r\theta}$ BENEATH CENTER OF CIRCLE)
IRREGULAR LOAD		VERTICAL	COMPUTED FROM INFLUENCE CHART OF FIGURE 10

ASSUMED CONDITIONS APPLIED LOADS ARE PERFECTLY FLEXIBLE FOUNDATION IS SEMI-INFINITE ELASTIC ISOTROPIC SOLID.

Table 2

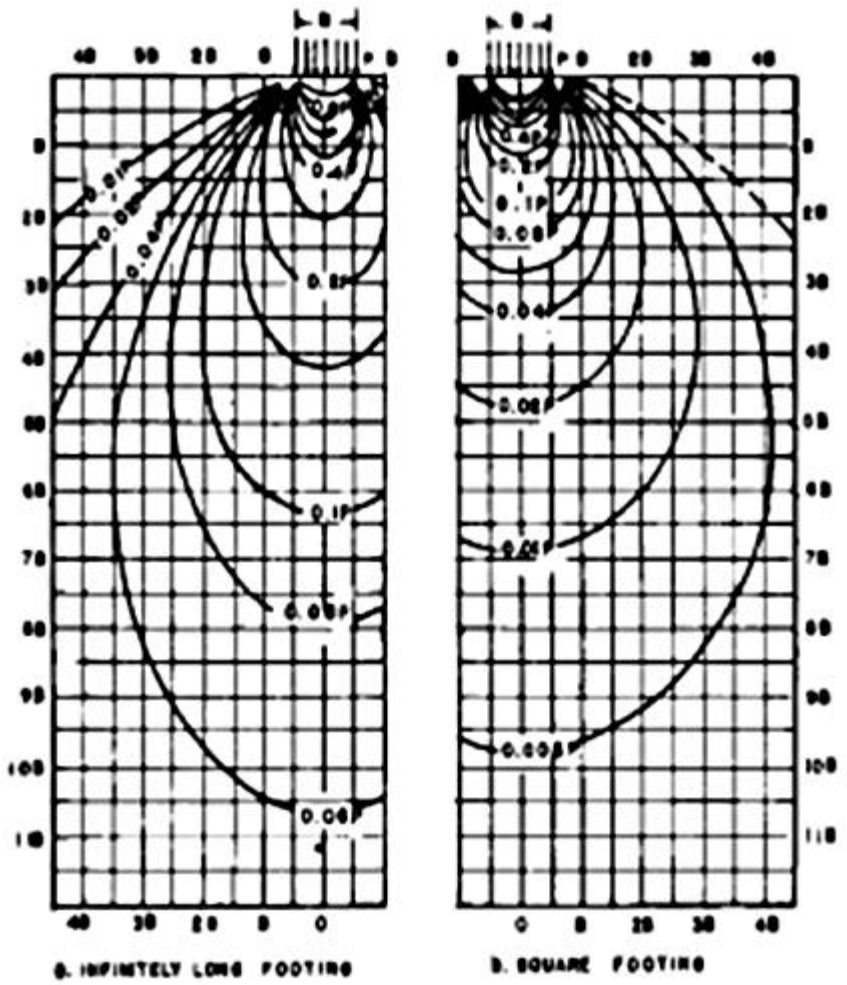
Formulas for stresses in semi-infinite elastic foundation

LOADING CONDITION	STRESS DIAGRAM	STRESS COMPONENT	EQUATION
EMBANKMENT LOADS OF INFINITE LENGTH			
UNIFORM STRIP LOAD		VERTICAL	$\sigma_z = \frac{p}{\pi}\left[\alpha + \sin\alpha\cdot\cos(\alpha+2\gamma)\right]$
		HORIZONTAL	$\sigma_x = \frac{p}{\pi}\left[\alpha - \sin\alpha\cdot\cos(\alpha+2\gamma)\right]$
		SHEAR	$\tau_{xz} = \frac{p}{\pi}\left[\sin\alpha\cdot\sin(\alpha+2\gamma)\right]$
TRIANGULAR LOAD		VERTICAL	$\sigma_z = \frac{p}{\pi}\left[\frac{1}{b}\alpha + \frac{z}{b}(\alpha+\beta)\right]$
		HORIZONTAL	$\sigma_x = \frac{p}{\pi}\left[\frac{1}{b}\alpha + \frac{z}{b}\sin 2\beta + \frac{2z}{b}\log_e\frac{R_2}{R_0} + \frac{2z}{b}\log_e\frac{R_1}{R_0}\right]$
		SHEAR	$\tau_{xz} = \frac{pz}{\pi}\left[\frac{1}{b}\cdot\frac{\beta}{b}\right]$
SLOPE LOAD		VERTICAL	$\sigma_z = \frac{p_0}{\pi}\left[\chi\beta + z\right]$
		HORIZONTAL	$\sigma_x = \frac{p_0}{\pi}\left[\chi\beta - z - z\log_e R\right]$
		SHEAR	$\tau_{xz} = \frac{p_0}{\pi}\cdot z\beta$
TERRACE LOAD		VERTICAL	$\sigma_z = \frac{p}{\pi_0}\left[\omega\beta + \chi\alpha\right]$
		HORIZONTAL	$\sigma_x = \frac{p}{\pi_0}\left[\omega\beta + \chi\alpha + 2z\log_e\frac{R_2}{R_1}\right]$
		SHEAR	$\tau_{xz} = \frac{p}{\pi_0}\cdot z\alpha$
SEMI - INFINITE UNIFORM LOAD		VERTICAL	$\sigma_z = \frac{p}{\pi}\left[\beta + \frac{xz}{R^2}\right]$
		HORIZONTAL	$\sigma_x = \frac{p}{\pi}\left[\beta - \frac{xz}{R^2}\right]$
		SHEAR	$\tau_{xz} = -\frac{p}{\pi}\cdot\sin^2\beta$

ASSUMED CONDITIONS: APPLIED LOADS ARE PERFECTLY FLEXIBLE; FOUNDATION IS SEMI-INFINITE ELASTIC ISOTROPIC SOLID.

Table 2 (continued)

Formulas for stresses in semi-infinite elastic foundation



SQUARE FOOTING

GIVEN

FOOTING SIZE = 20' x 20'

UNIT PRESSURE P = 2 TSF

FIND

PROFILE OF STRESS INCREASE BENEATH CENTER OF FOOTING DUE TO APPLIED LOAD

B = 20' P = 2 TSF

$\frac{z}{B}$ (FT)	$\frac{z}{B}$	σ_z TSF
10	0.5	0.70 x 2 = 1.4
20	1	0.38 x 2 = 0.76
30	1.5	0.19 x 2 = 0.38
40	2.0	0.12 x 2 = 0.24
50	2.5	0.07 x 2 = 0.14
60	3.0	0.05 x 2 = 0.10

Figure 3

Stress contours and their application

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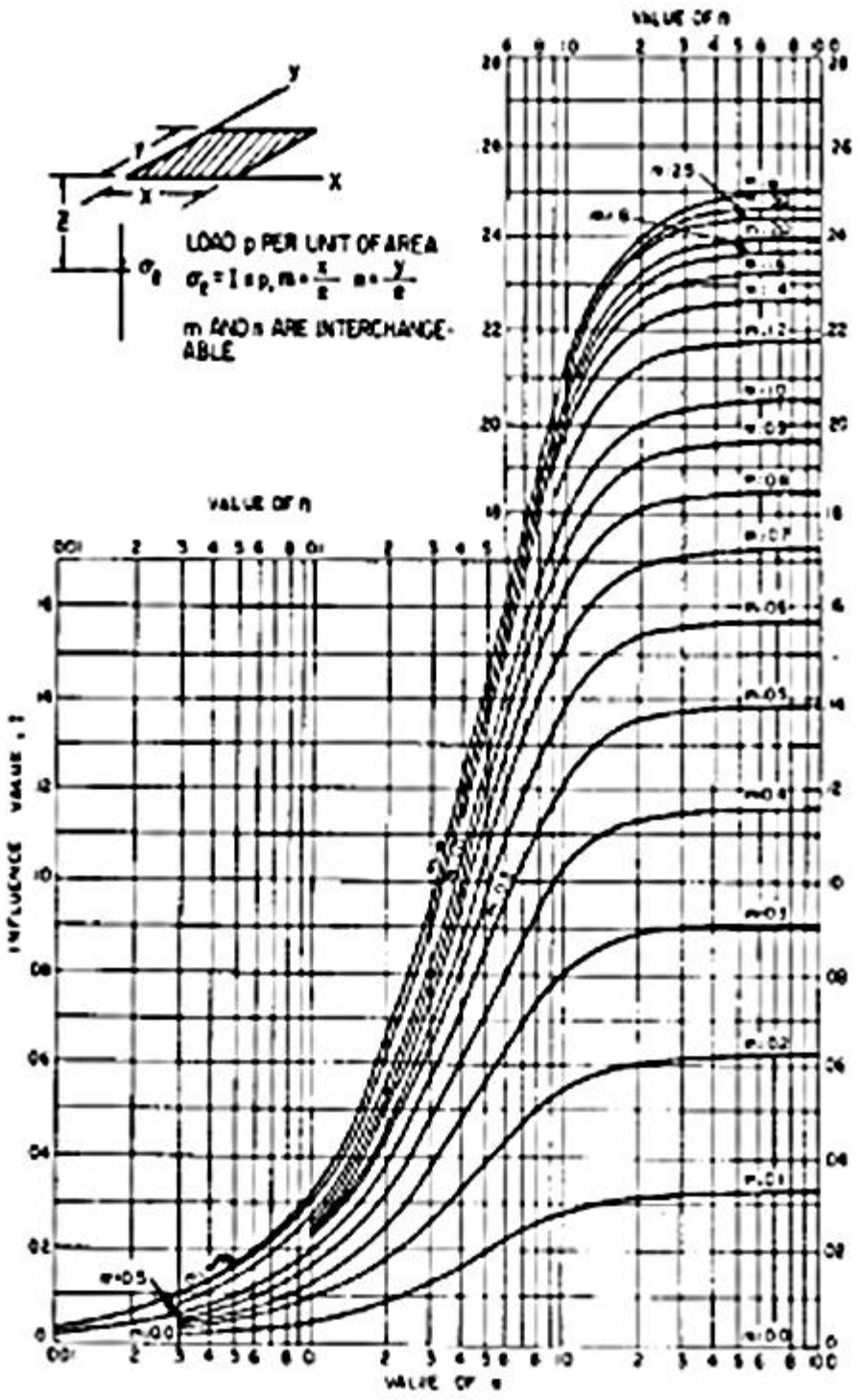


Figure 4

Influence value for vertical stress beneath a corner

of a uniformly loaded rectangular area

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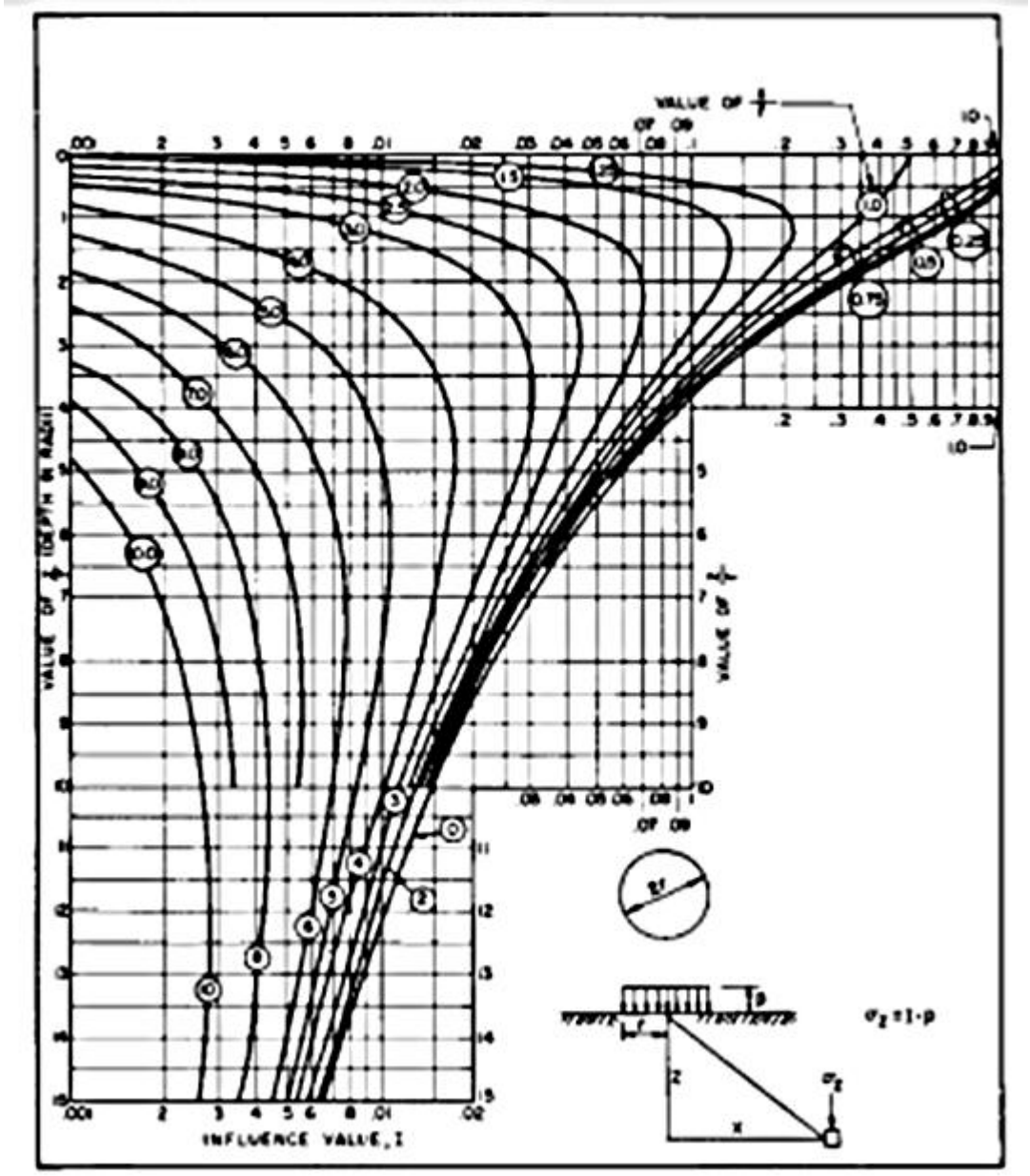


Figure 5

Influence value for vertical stress beneath a corner

of a uniformly loaded rectangular area

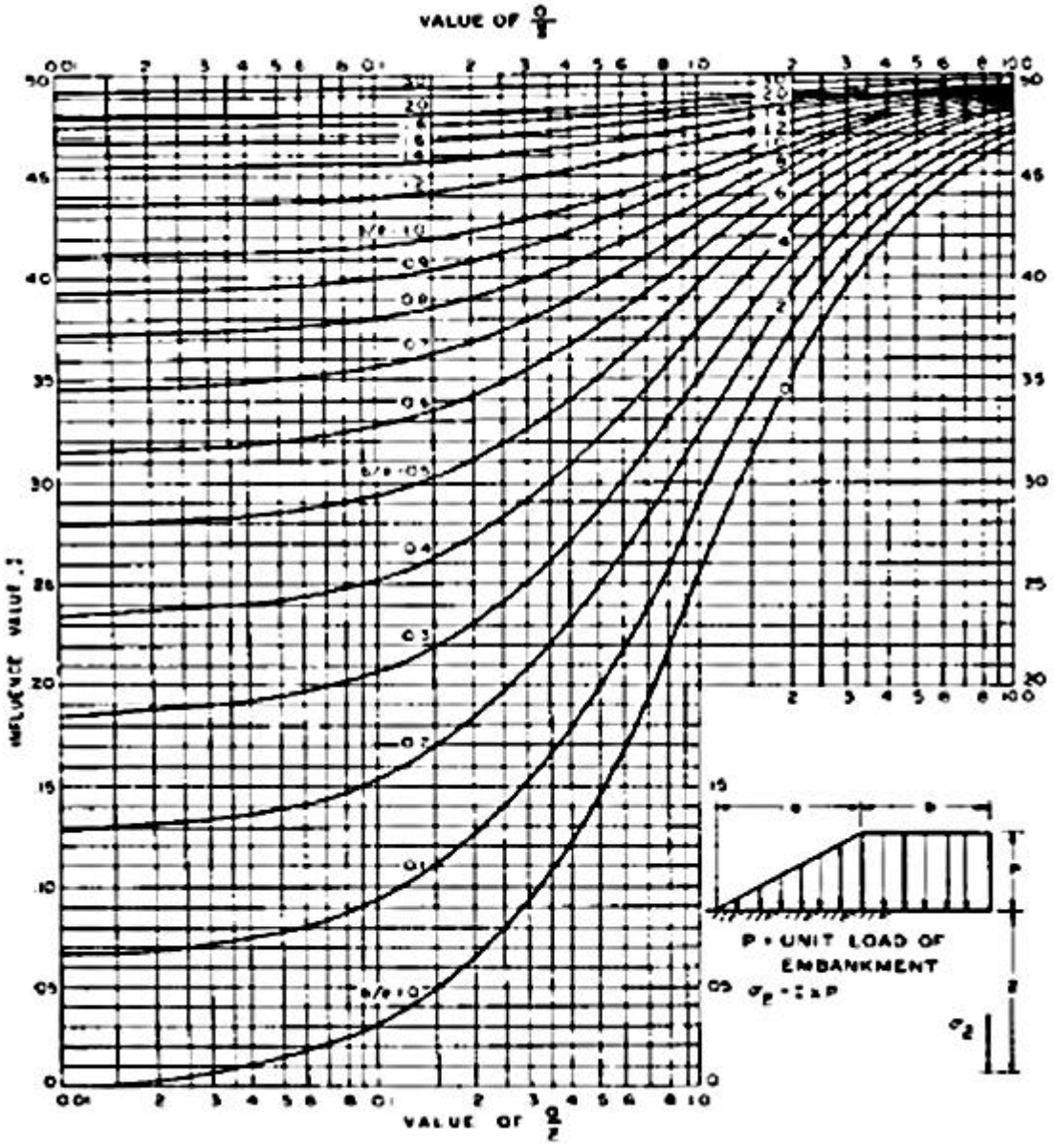


Figure 6

Influence value for vertical stress under embankment load of infinite length

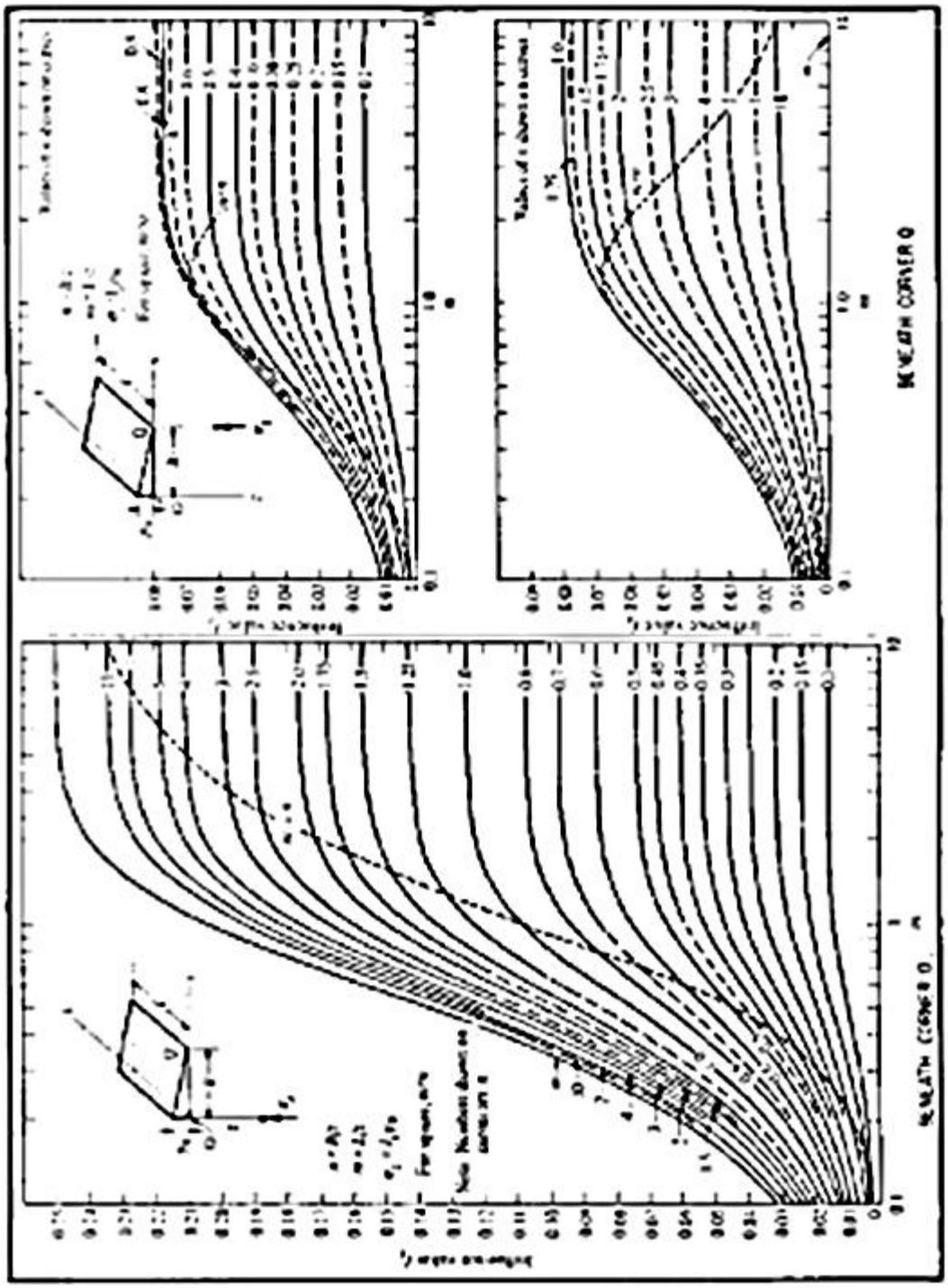


Figure 7

Influence value for vertical stress beneath triangular load

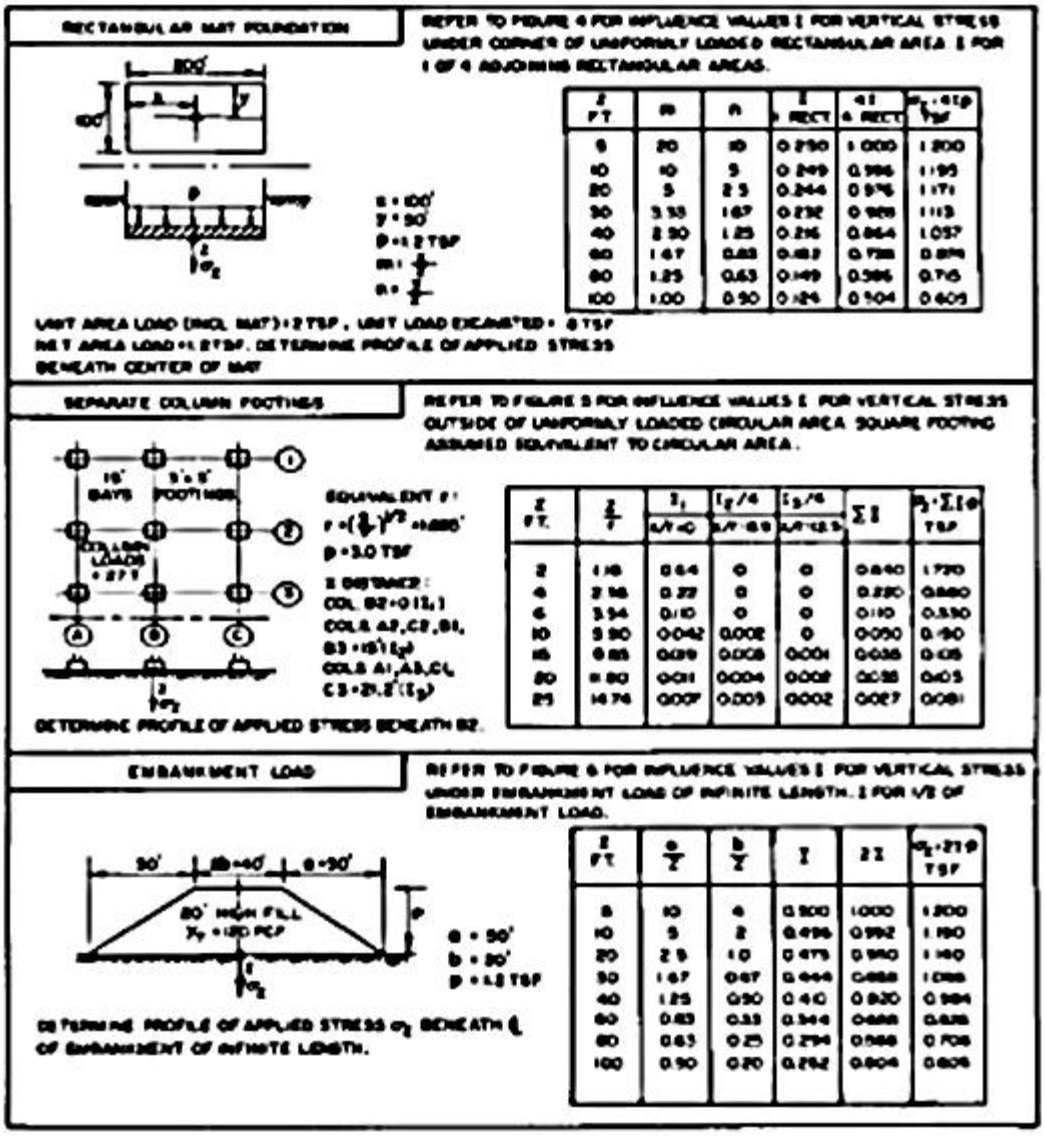


Figure 8

Examples of computation of vertical stress

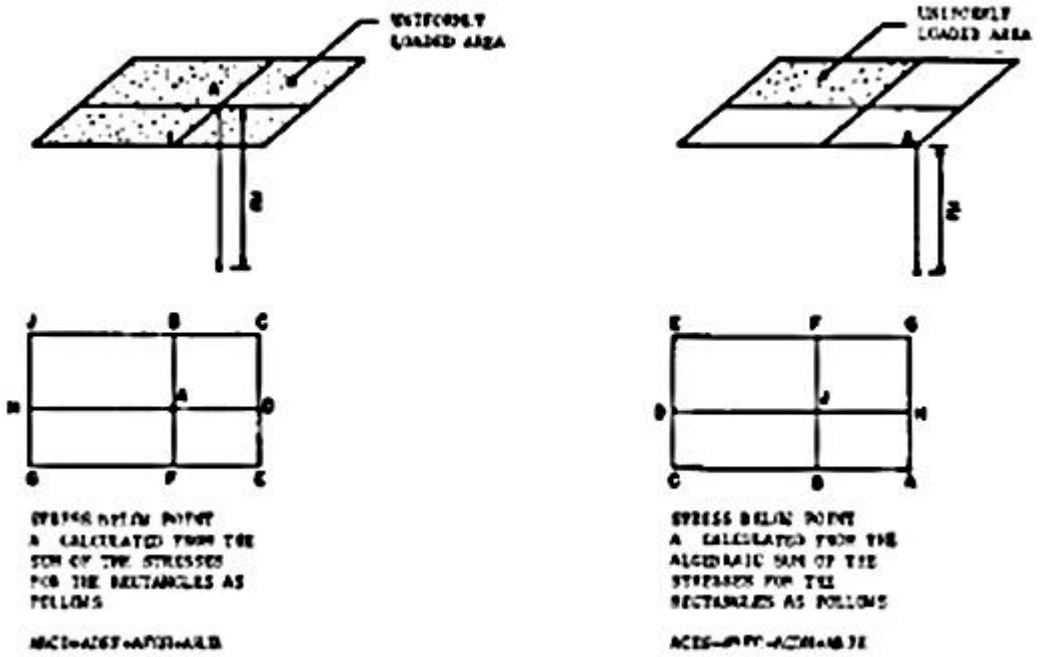


Figure 9

Determination of stress below corner of uniformly loaded rectangular area

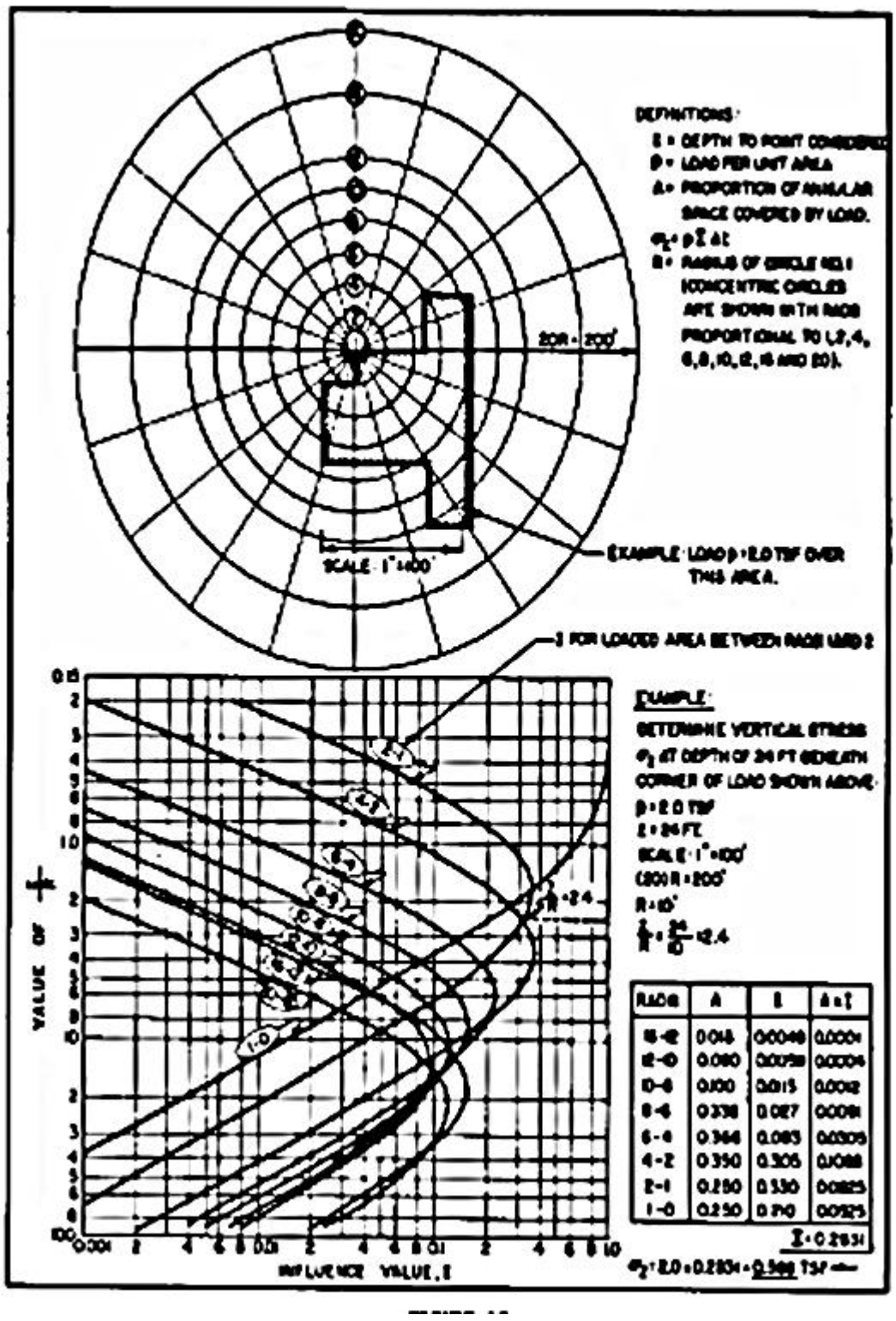


Figure 10

Influence chart for vertical stress beneath triangular load

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(4) See the bottom chart of Figure 10 for influence values for stresses at various depths produced by the loads within each annular space. The product I x A multiplied by the load intensity equals vertical stress.

(5) To determine a profile of vertical stresses for various depths beneath a point, the target need not be redrawn. Obtain influence values for different ordinates Z/R from the influence chart.

3.1.3.5. HORIZONTAL STRESSES. Elastic analysis is utilized to determine horizontal stresses on unyielding walls from surcharge loads and pressures on rigid buried structures. (See basic formulas for simple loads in Figure 2.)

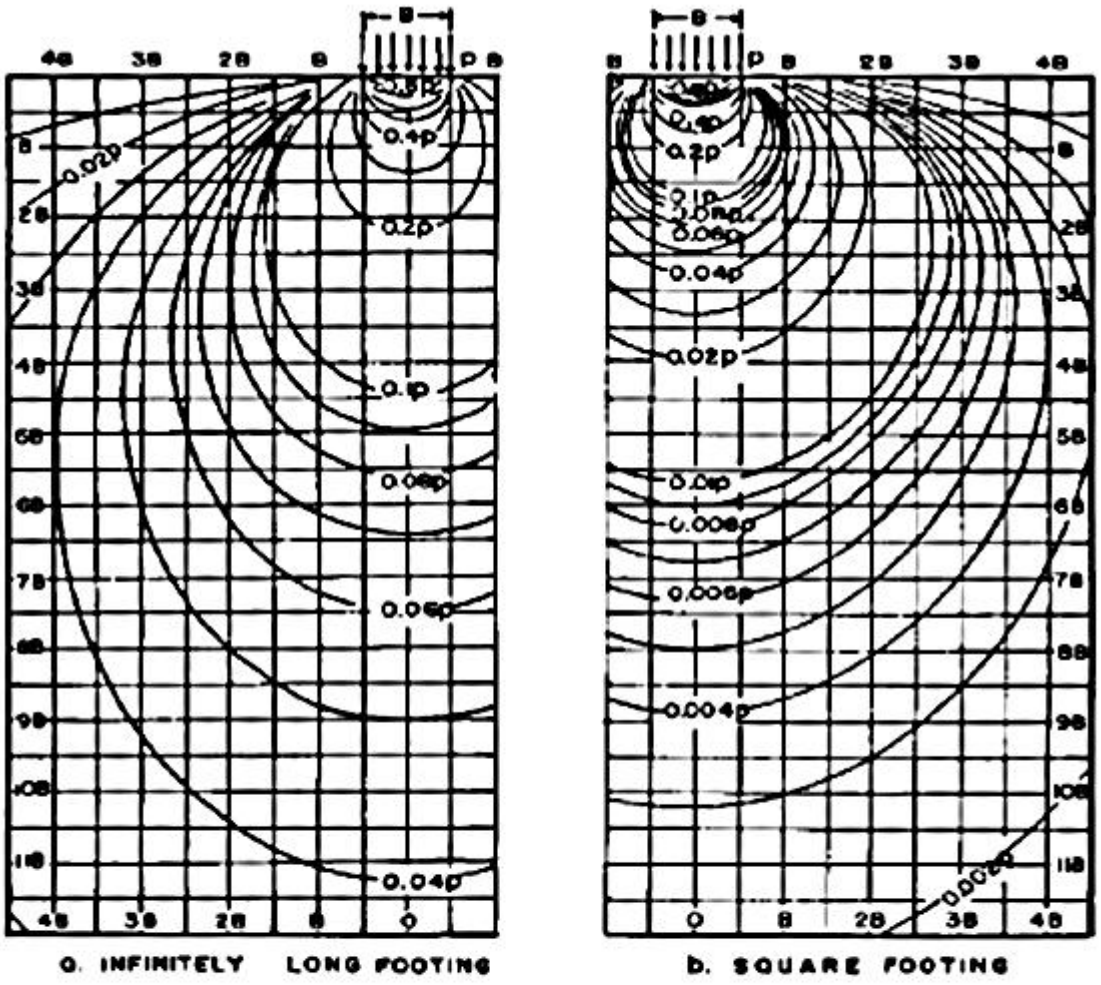
3.1.3.6 SHEAR STRESSES. Elastic solutions generally are not applicable when shear stresses are critical, as in stability problems. To determine if a stability analysis is required, determine the maximum shear stress from elastic formulas and compare this stress with the shear strength of the soil. For embankment loads in Figure 2, maximum shear stress in the foundation is exactly or approximately equal to p/[pi] depending upon the shape of the load and point in question. If the maximum shear stress equals shear strength, plastic conditions prevail at some point in the foundation soil and if the load is increased, a larger and larger portion of the foundation soil passes into plastic equilibrium. In this case, failure is possible and overall stability must be evaluated.

3.2 LAYERED OR ANISOTROPIC FOUNDATIONS. Actual foundation conditions differ from the homogeneous isotropic, semi-infinite mass assumed in the Boussinesq expressions. The modulus of elasticity usually varies from layer to layer, and soil deposits frequently are more rigid in the horizontal direction than in the vertical.

3.2.1 WESTERGAARD ANALYSIS. The Westergaard analysis is based on the assumption that the soil on which load is applied is reinforced by closely spaced horizontal layers which prevent horizontal displacement. The effect of the Westergaard assumption is to reduce the stresses substantially below those obtained by the

Boussinesq equations. The Westergaard analysis is applicable to soil profiles consisting of alternate layers of soft and stiff materials, such as soft clays with frequent horizontal layers of sand having greater stiffness in the horizontal direction. Figures 11, 12 and 13 can be used for calculating vertical stresses in Westergaard material for three loading conditions. Computations for Figures 11, 12, and 13 are made in a manner identical to that for Figures 3, 4, and 7, which are based on the Boussinesq equations. For illustration see Figure 8.

3.2.2 LAYERED FOUNDATIONS. When the foundation soil consists of a number of layers of substantial thickness, having distinctly different elastic properties, the vertical and other stresses are markedly different from those obtained by using the Boussinesq equation. (See Figure 14 for influence values of vertical stresses in a two-layer foundation with various ratios of modulus of elasticity. See Figure 15 for an example.).



EXAMPLE:

FIND THE PRESSURE INCREASE DUE TO A STRIP FOOTING OF WIDTH B, AT A POINT LOCATED 6B BELOW ITS BASE AND 3B FROM THE CENTER OF THE FOOTING. SURFACE LOAD ON THE FOOTING IS P PER-UNIT AREA. FROM THE LEFT PANEL, PRESSURE INCREASE = 0.05 P

Figure 11

Vertical stress contours for square and strip footings

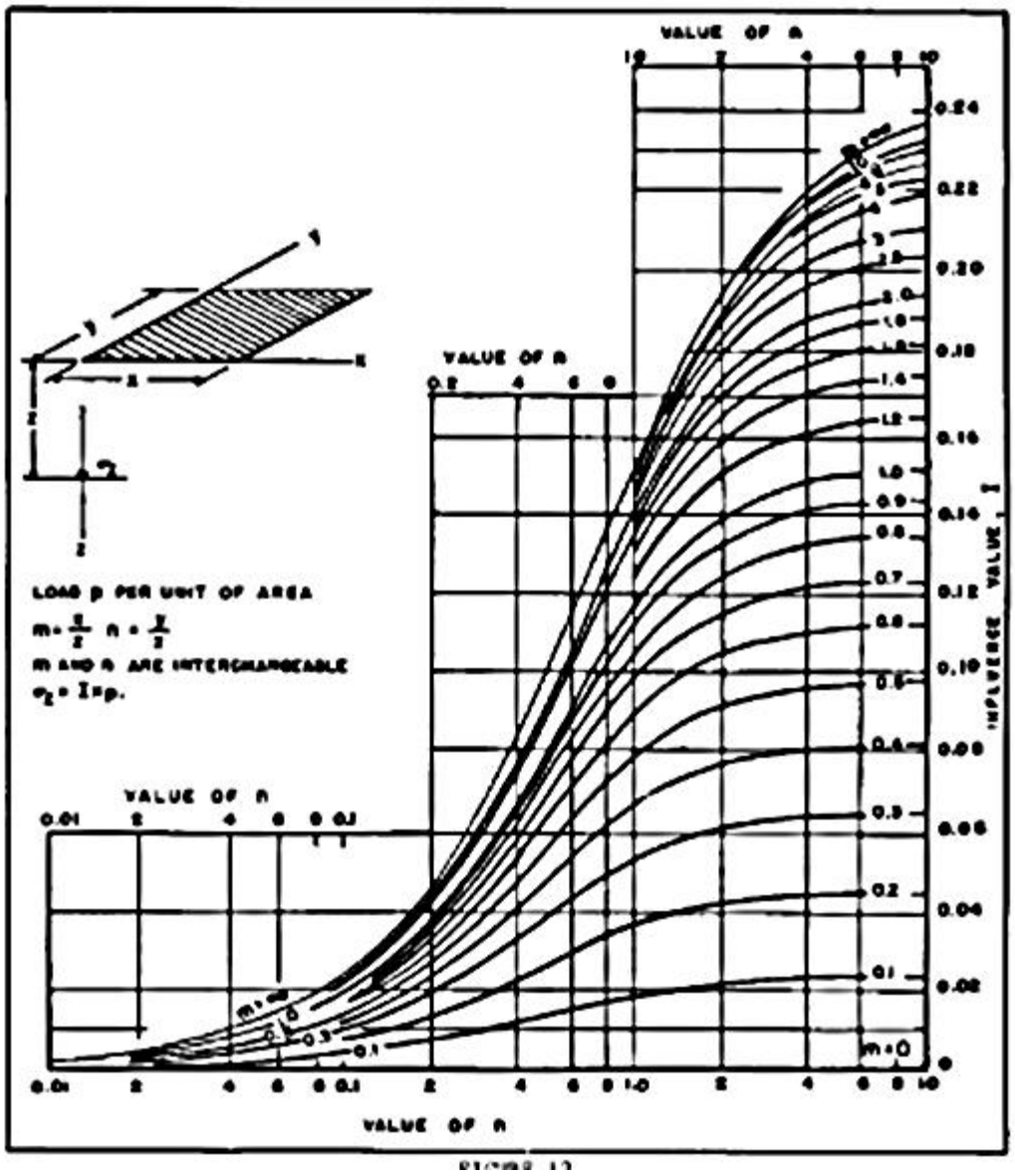


Figure 12

Influence value for vertical stress beneath a corner

of a uniformly loaded rectangular area

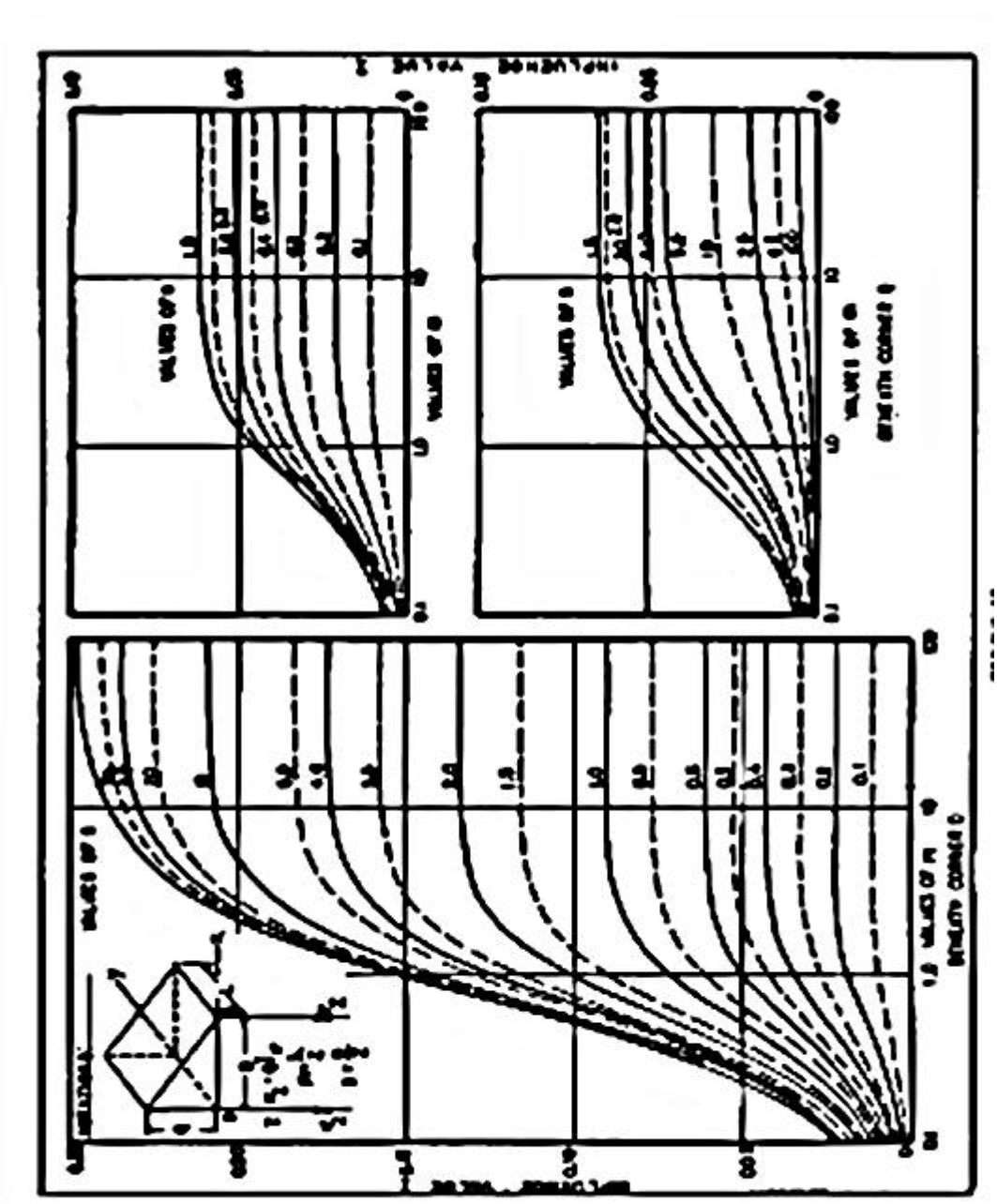


Figure 13

Influence value for vertical stress beneath triangular load

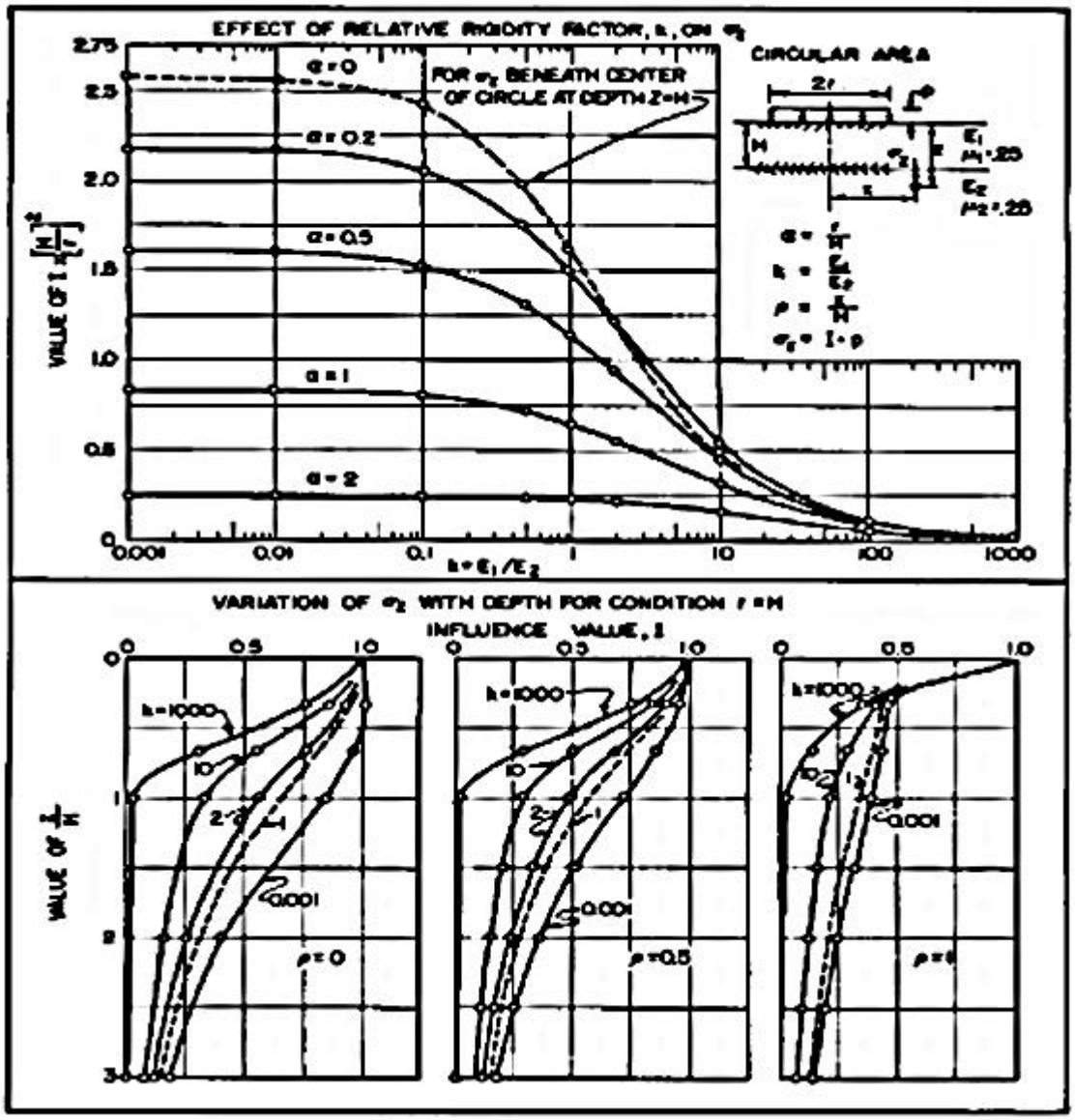
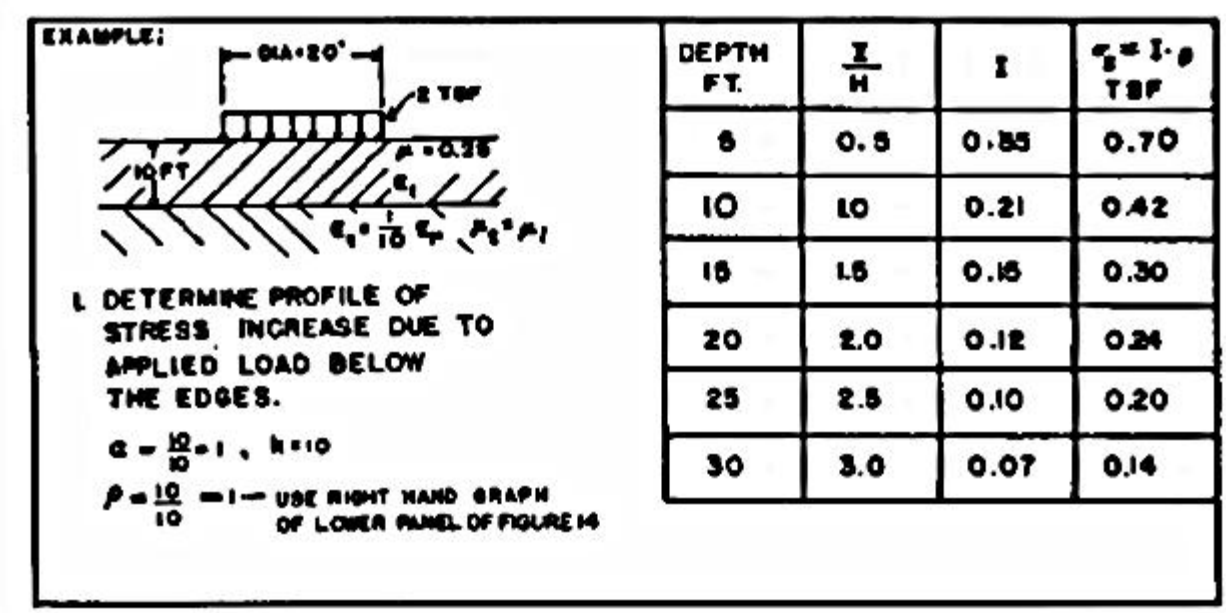


Figure 14

Influence values for vertical stresses beneath

uniformly loaded circular area (two layer foundation)



DEPTH FT.	$\frac{Z}{H}$	I	$\sigma_z = I \cdot q$ TSF
5	0.5	0.35	0.70
10	1.0	0.21	0.42
15	1.5	0.15	0.30
20	2.0	0.12	0.24
25	2.5	0.10	0.20
30	3.0	0.07	0.14

Figure 15

Stress profile in a two layer soil mass

3.2.2.1 RIGID SURFACE LAYER OVER WEAKER UNDERLYING LAYER. If the surface layer is the more rigid, it acts as a distributing mat and the vertical stresses in the underlying soil layer are less than Boussinesq values.

3.2.2.2 WEAKER SURFACE LAYER OVER STRONGER UNDERLYING LAYERS. If the surface layer is less rigid than the underlying layer, then vertical stresses in both layers exceed the Boussinesq values. Use these influence diagrams to determine vertical stress distribution for settlement analysis involving a soft surface layer underlain by stiff material.

3.2.3 CRITICAL DEPTH. If there is no distinct change in the character of subsurface strata within the critical depth, elastic solutions for layered foundations need not be considered. Critical depth is the depth below the foundation within which soil compression contributes significantly to surface settlements. For fine-grained compressible soils, the critical depth extends to that point where applied stress decreases to 10 percent of effective overburden pressure. In coarse-grained material

critical depth extends to that point where applied stress decreases to 20 percent of effective overburden pressure.

3.3 RIGID LOADED AREA. A rigid foundation must settle uniformly. When such a foundation rests on a perfectly elastic material, in order for it to deform uniformly the load must shift from the center to the edges, thus resulting in a pressure distribution which increases toward the edges (see Figure 16). This is the case for clays. In the case of sands, the soil near the edges yields because of the lack of confinement, thus causing the load to shift toward the center.

3.4 STRESSES INDUCED BY PILE LOADS. Estimates of the vertical stresses induced in a soil mass by an axially loaded pile are given in Figure 17 for both friction and end-bearing piles.

4. SHALLOW PIPES AND CONDUITS

4.1 GENERAL. Pressures acting on shallow buried pipe and conduits are influenced by the relative rigidity of the pipe and surrounding soil, depth of cover, type of loading, span (maximum width) of structure, method of construction, and shape of pipe. This section describes simple procedures for determining pressures acting on a conduit in compressible soil for use in conduit design. For detailed analysis and design procedures for conduits in backfilled trenches and beneath embankments, consult one of the following:

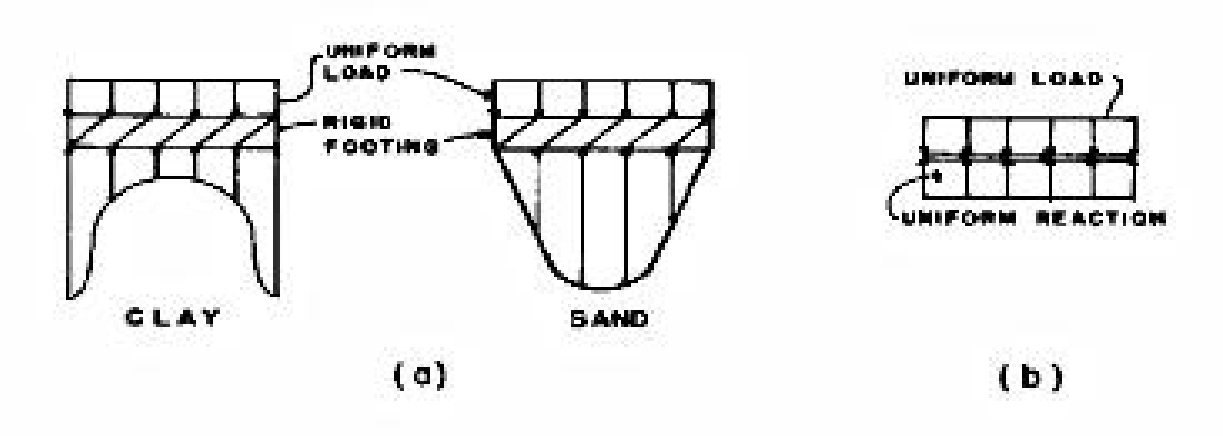


Figure 16
Contact pressure under (a) rigid footings and
(b) flexible foundation on an elastic half space

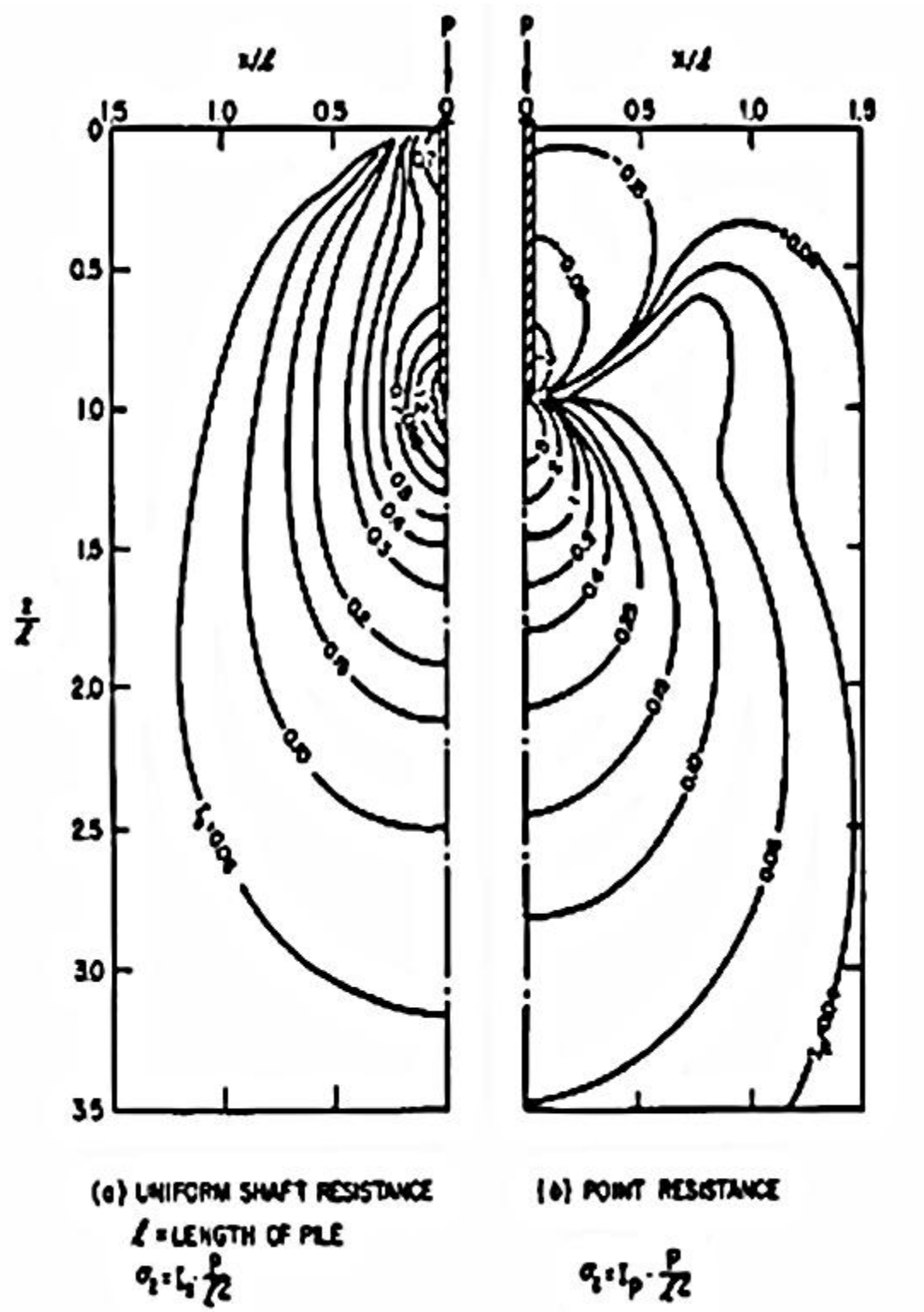


Figure 17

Influence values for vertical stresses around a pile in an elastic solid

Buried Structures, by Watkins; *Design and Construction of Sanitary and Storm Sewers*, by the American Society of Civil Engineers; *Handbook of Drainage and Construction Products*, by Armco Drainage and Metal Products, Inc.; *Engineering Handbook, Structural Design*, by the U.S. Department of Agriculture, Soil Conservation Service; *Concrete Pipe Design Manual*, by American Concrete Pipe Association; or *CANDE User Manual*, by Katona and Smith.

4.2 RIGID PIPE. Pipes made from precast or cast-in-place concrete, or cast iron are considered rigid pipes.

4.2.1 VERTICAL LOADS.

4.2.1.1 DEAD LOAD. Vertical soil pressure estimates for dead loads are obtained as follows:

EQUATION: $W = C_w \times \upsilon \times B^2$

where

W = total dead load on the conduit per unit length of conduit
C_w = correction coefficient; function of trench depth to width ratio, angle of trench side slopes, friction angle of backfill and trench sides, bedding conditions
B = width of trench at level of top of pipe, or pipe outside diameter if buried under an embankment
υ = unit weight of backfill
Dead load pressure, $P_{DL} = W/B$

4.2.1.1.1 EMBANKMENT FILL. Use Figure 18a to determine embankment dead load. For soils of unit weight other than 100 pcf, adjust proportionately; e.g., for υ = 120 pcf, multiply chart by 1.20.

4.2.1.1.2 TRENCH BACKFILL. Use Figure 18b to determine values of C_w.

4.2.1.1.3 JACKED OR DRIVEN INTO PLACE. Use Figure 18c for C_w. This diagram may also be used for jacked tunnels.

4.2.2 LIVE LOAD. Vertical pressure due to surface load, P_{LL} is calculated by using Figure 2. Impact factor is included in the live load if it consists of traffic load. For example, an H-20 truck loading consists of two 16,000 lb. loads applied to two 10- by 20-inch areas. One of these loads is placed over the point in question, the other is 6 feet away. The vertical stresses produced by this loading including the effect of impact are shown in Figure 19 for various heights of cover.

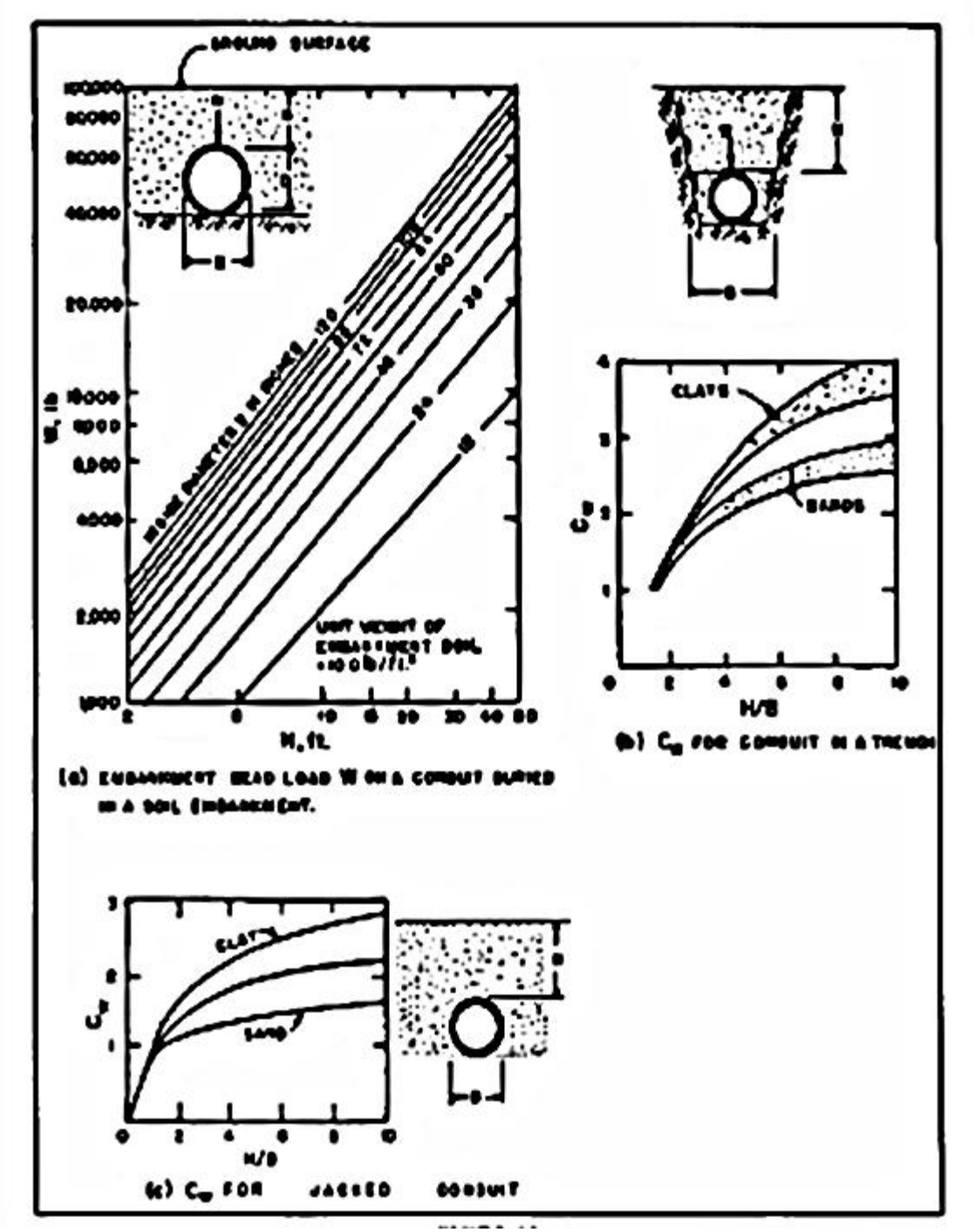
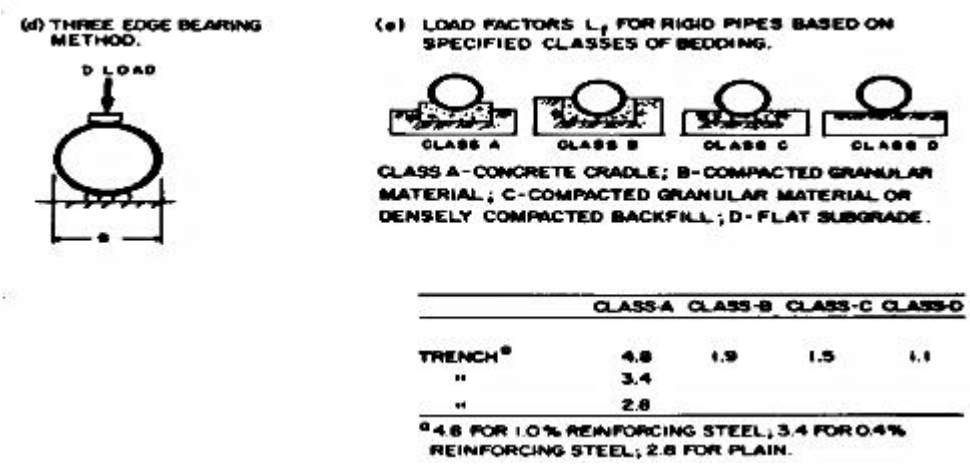


Figure 18

Backfill coefficients, embankment loads, and load factors for rigid conduits



	CLASS A	CLASS B	CLASS C	CLASS D
TRENCH[a]	4.8	1.9	1.5	1.1
"	3.4			
"	2.8			

[a] 4.8 FOR 1.0% REINFORCING STEEL; 3.4 FOR 0.4% REINFORCING STEEL; 2.8 FOR PLAIN.

Figure 18 (continued)

Backfill coefficients, embankment loads, and load factors for rigid conduits

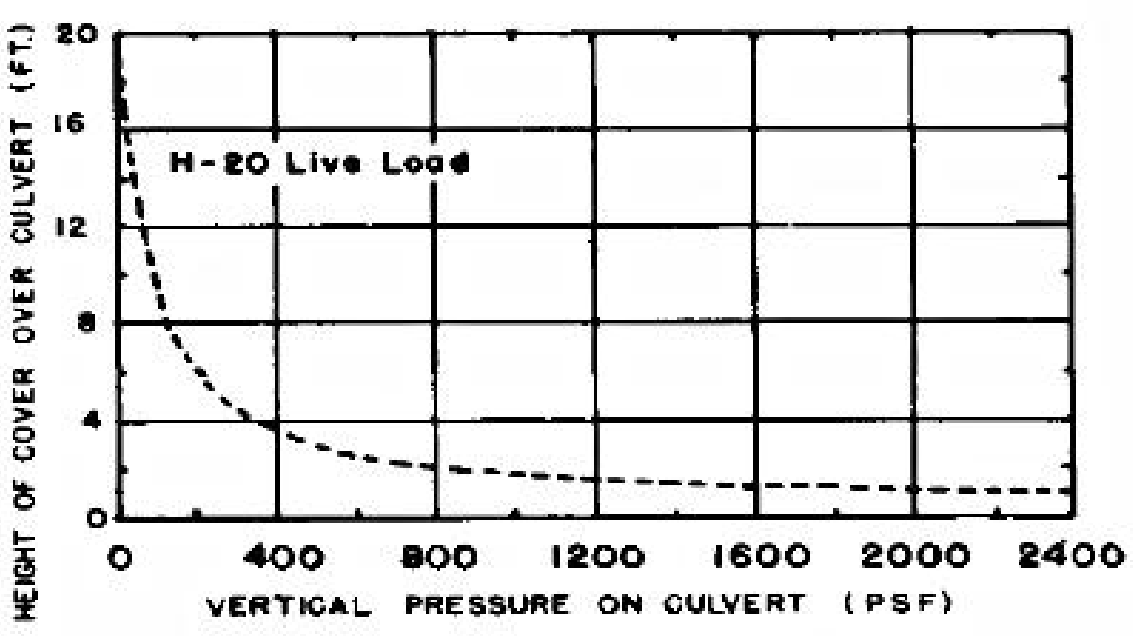


Figure 19

Vertical pressure on culvert versus height of cover

4.2 DESIGN OF RIGID CONDUIT. To design a rigid conduit, the computed loads (dead and live) are modified to account for bedding conditions and to relate maximum allowable load to the three-edge bearing test load D. (See Figure 18d.) See ASTM C76, Reinforced Concrete Culvert, Storm Drain, and Sewer Pipe, for test standards for D load. Bedding conditions for pipes in trenches may be accounted for by use of a load factor, L_f. Determine L_f from Figure 18e. Determine D from the following equation:

$$D_{0.01} = (P_{DL} + P_{LL}+LL,) (N/L_f)$$

where

$D_{0.01}$ = Allowable load in lb/ft of length of conduit per foot of inside diameter for a crack width of 0.01"
L_f = load factor
N = safety factor (usually 1.25)

With the specified D load, the supplier is able to provide adequate pipe. The soil pressure against the sides of a pipe in an embankment significantly influences the resistance of the pipe to vertical load. The load factor for such cases considers not only pipe bedding, but also pipe shape, lateral earth pressure, and the ratio of total lateral pressure to total vertical pressure.

4.3 FLEXIBLE STEEL PIPE. Corrugated or thin wall smooth steel pipes are sufficiently flexible to develop horizontal restraining pressures approximately equal to vertical pressures if backfill is well compacted. Vertical exterior pressure acting at the top of the pipe may range from pressures exceeding overburden pressure in highly compressible soils to much less than the overburden pressure in granular soils because of the effect of "arching", in which a portion of the overburden pressure is supported by the surrounding soil.

4.3.1 VERTICAL LOADS.

4.3.1.1 DEAD LOAD. For flexible pipe, the dead load pressure is simply the height of the column of soil above the conduit times the unit weight of the backfill, as follows:.

$$P_{DL} = \upsilon \times H$$

4.3.1.2 LIVE LOAD. Computed by Boussinesq equations for rigid pipes.

4.3.1.3 PRESSURE TRANSFER COEFFICIENT. The dead load and live load pressures are modified by pressure transfer coefficient, C_p to yield apparent pressure, P, to be used in design.

$$P = C_P (P_{LL} + P_{DL})$$

See Figure 20 for the values of C_P.

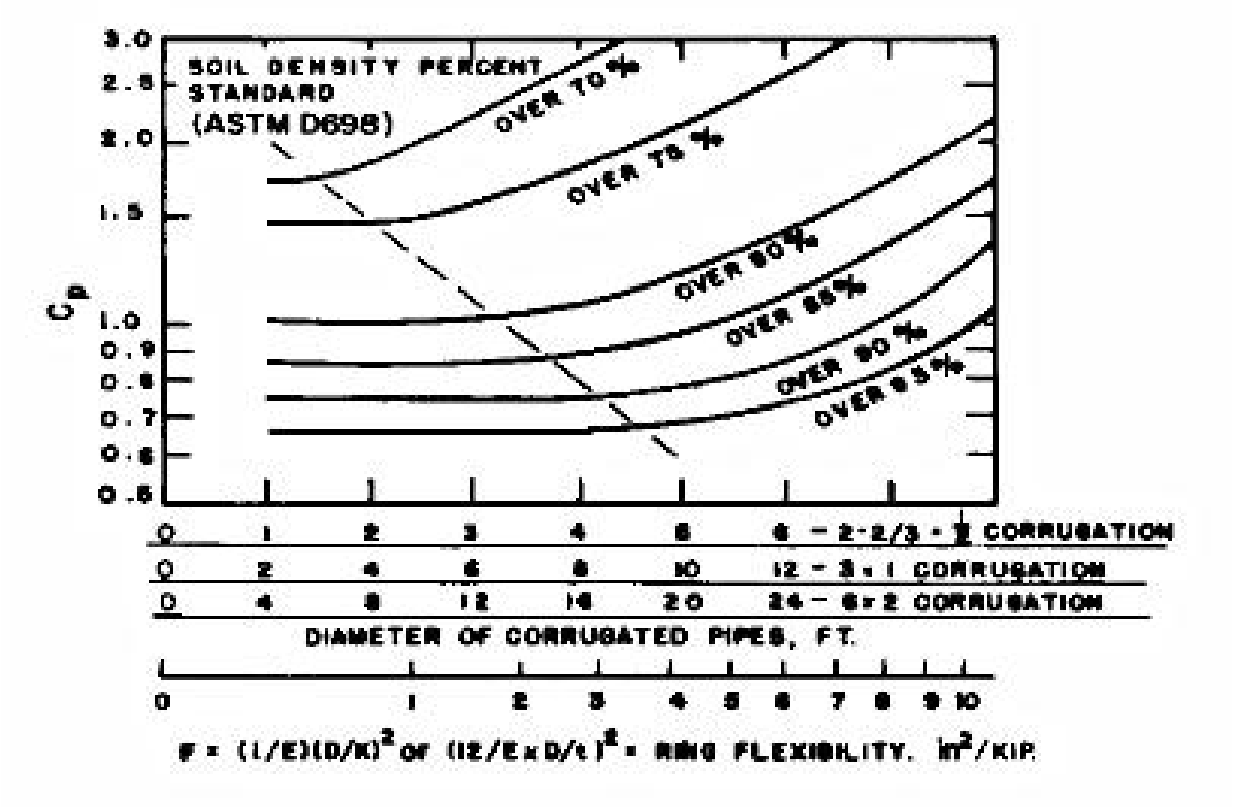


Figure 20

Pressure transfer coefficients for corrugated flexible conduits as a function of standard soil density and ring flexibility or diameter and corrugation depth

4.3.2 INITIAL DESIGNS. Use the following design procedures:

4.3.2.1 DETERMINE apparent ring compression stress of the pipe:

Apparent ring comp. stress = PD/2A

where

P = apparent vertical soil pressure on top of conduit

D = outside diameter of conduit

A = cross-sectional area of the wall per unit length of conduit

4.3.2.2 EQUATE apparent ring compression stress to allowable ring compression strength to determine required cross-sectional wall area, A, per unit length of pipe:

Allowable ring comp. strength = S_y/F_S

$A = (PDS_y)/2F_S$

where

S_y = yield point strength of the steel (typically 33 to 45 ksi)
F_S = safety factor (usually 1.5 to 2)

4.3.2.3 SELECT appropriate pipe size to provide the minimum cross-sectional wall area A as determined above.

4.3.2.4 CHECK RING DEFLECTION so that it does not exceed 5% of the nominal diameter of the pipe. Ring deflection Y, is governed by the total soil pressure $P_V = P_{DL}+P_{LL}$, diameter D, moment of inertia I, modulus of elasticity of conduit E, and soil modulus E'. Generally, ring deflection does not govern the design. See Figure 21 for an example.

4.3.2.5 THE HANDLING FACTOR is the maximum flexibility beyond which ring is easily damaged. Pipe design must consider limiting the Handling Factor to such typical values as D^2/EI = 0.0433 in/lb for 2-2/3 x 1/2 corrugation and 0.0200 in/lb for 6 x 2 corrugation.

4.3.3 SOIL PLACEMENT. Great care must be exercised in soil placement. Ring deflection and external soil pressures are sensitive to soil placement. If a loose soil blanket is placed around the ring and the soil is carefully compacted away from it, soil pressure is reduced considerably.

4.3.4 DESIGN OF FLEXIBLE STEEL PIPE. For analysis and design procedures for large size flexible pipe of non-circular cross section, refer to the technical literature.

4.4 CONDUITS BENEATH EMBANKMENTS OF FINITE WIDTH. Design of culverts and conduits beneath narrow-crested embankments must consider the effect of the embankment base spread and settlement on the pipe.

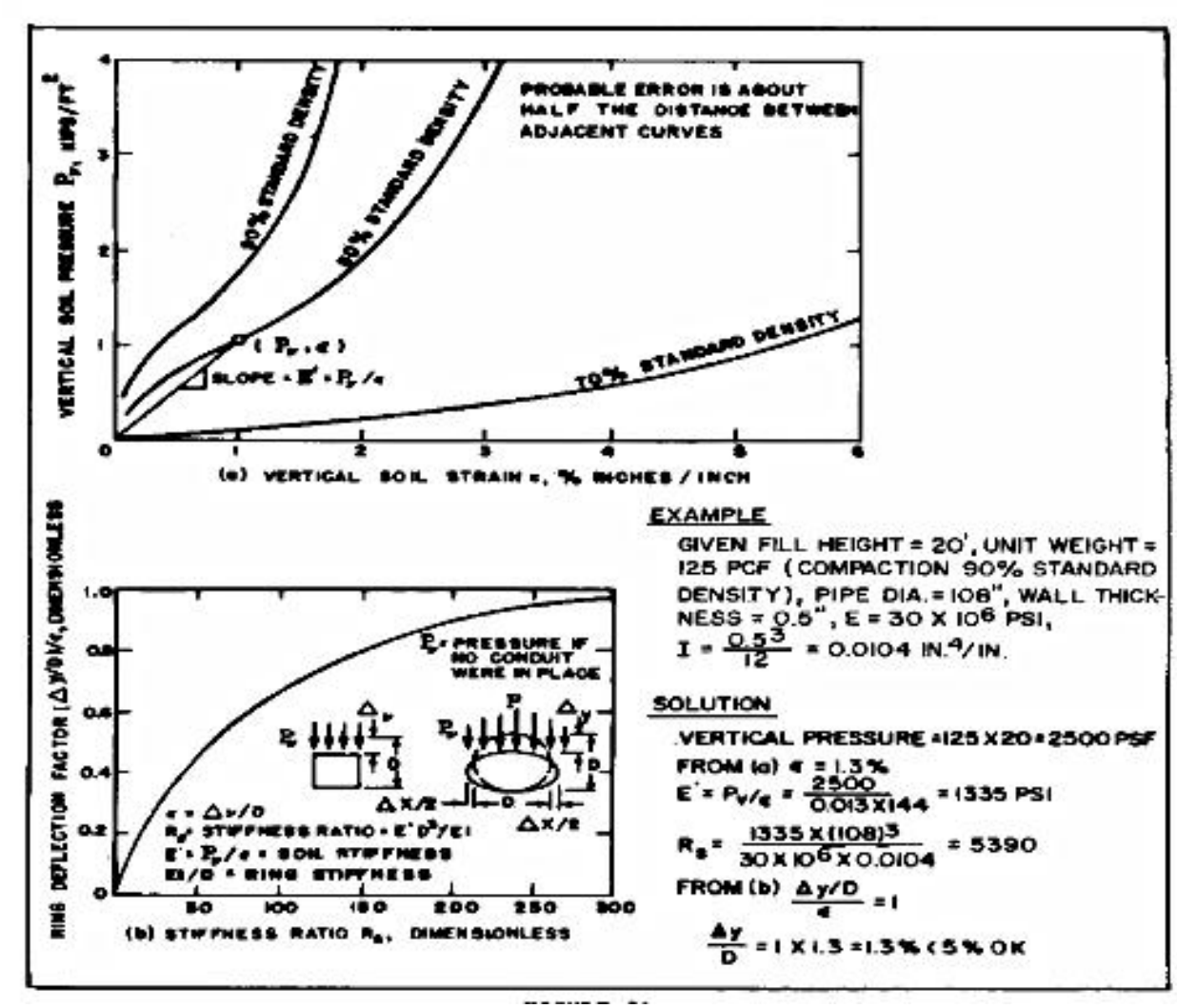


Figure 21

Example of ring deflection

4.4.1 LONGITUDINAL EXTENSION. The maximum horizontal strain of a conduit beneath an embankment or earth dam occurs under the center of the fill. Maximum strain depends on the ratios b/h, b/d, and the average vertical strain in the foundation beneath center of the fill. (See Figure 22 for the definitions and the relationship between vertical strain and horizontal strain.)

4.4.2 JOINT ROTATION. Besides the horizontal extension of the conduit, additional joint opening may occur at the bottom of the pipe because of settlement under the embankment load. For concrete pipe in sections about 12 feet long, compute additional joint opening due to settlement by the equation below:

$$\text{Opening} = \Delta cr/b$$

where

Δ = settlement of base of pipe at embankment centerline (in)
b = embankment base width (in)
c = constant, varying from 5 for uniform foundation conditions to 7 for variable foundation conditions
r = pipe radius (in)

4.4.3 PIPE SELECTION. Compute total settlement below embankment. From this value, compute maximum joint opening at pipe mid-height as above. Add to this opening the spread at the top or bottom of the pipe from joint rotation computed from the equation above. Specify a pipe joint that will accommodate this movement and remain watertight. If the joint opening exceeds a safe value for precast concrete pipe, consider cast-in-place conduit in long sections with watertight expansion joints. Corrugated metal pipe is generally able to lengthen without rupture, but it may not be sufficiently corrosion resistant for water retention structures.

4.5 LONG SPAN METAL CULVERTS. The above methods are not applicable to very large, flexible metal culverts, i.e., widths in the range of 25 to 45 feet. For analysis and design procedures for these see the technical literature.

5. DEEP UNDERGROUND OPENINGS

5.1 GENERAL FACTORS. Pressures acting on underground openings after their completion depend on the character of the surrounding materials, inward movement permitted during construction, and restraint provided by the tunnel lining.

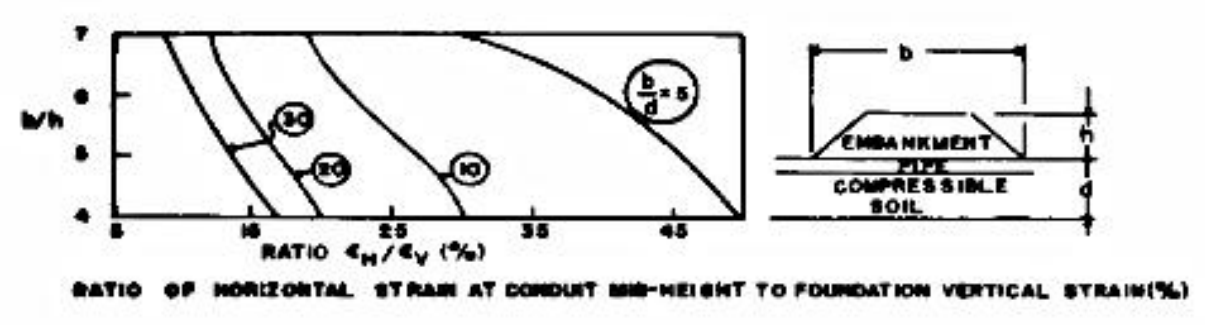


Figure 22

Conduits beneath embankments of finite width

5.2 OPENINGS IN ROCK. Stress analysis differs for two rock groups: sound, nonswelling rock that can sustain considerable tensile stresses, and fractured blocky, seamy, squeezing, or swelling rock.

5.2.1 SOUND ROCK. Determine stresses surrounding tunnels or openings in intact, isotropic rock, such as crystalline igneous types, or homogeneous sandstone and limestone, by elastic analyses. For these materials, stresses in rock surrounding spheroidal cavities are lower than those for tunnels with the same cross section. Use elastic analyses to determine the best arrangement of openings and pillars, providing supports as required at locations of stress concentrations. For initial estimates of roof pressure, Table 1 may be used.
be used.

5.2.2 BROKEN AND FRACTURED ROCK. Pressure on tunnels in chemically or mechanically altered rock must be analyzed by approximate rules based on experience.

5.2.3 SQUEEZING AND SWELLING ROCKS. Squeezing rocks contain a considerable amount of clay. The clay fraction may be from non-swelling kaolinite group or from highly swelling montmorillonite group. These rocks are preloaded clays and the squeezing is due to swelling. The squeeze is intimately related to an increase in water content and a decrease in shear strength.

5.3 LOADS ON UNDERGROUND OPENINGS IN ROCK

5.3.1 VERTICAL ROCK LOAD. Table 1 gives the height of rock above the tunnel roof which must be supported by roof lining.

5.3.2 HORIZONTAL PRESSURES. Determine the horizontal pressure P+a, on tunnel sides by applying the surcharge of this vertical rock load to an active failure wedge (see diagram in Table 1). Assume values of rock shear strength (see Chapter 3 for a range of values) on the active wedge failure plane, which allow for the fractured or broken character of the rock. Evaluate the possibility of movement of an active failure plane that coincides with weak strata or bedding intersecting the tunnel wall at an angle.

5.3.3 SUPPORT PRESSURES AS DETERMINED FROM ROCK QUALITY. As an alternate method of analysis, use empirical correlations in the technical literature to determine required support pressures as a function of rock mass quality "Q". The analysis incorporates rock quality designation (RQD) and various joint properties of the surrounding material, and is applicable for sound or fractured rock. Results may be used directly for evaluating type of roof or wall support required.

Rock Conditions	Rock Load H_p in Feet	Remarks
1. Hard and intact	Zero	Sometimes spalling or popping occurs.
2. Hard stratified or schistose	0 to 0.5 B	Light pressures.
3. Massive, moderately jointed	0 to 0.25 B	Load may change erratically from point to point.
4. Moderately blocky and seamy	0.25 B to 0.35 $(B+H_t)$	No side pressure.
5. Very blocky and seamy	0.35 to 1.10 $(B+H_t)$	Little or no side pressure.
6. Completely crushed but chemically intact	1.10 $(B+H_t)$	Considerable side pressure. Softening effect of seepage towards bottom of tunnel.
7. Squeezing rock, moderate depth	(1.10 to 2.10) $(B+H_t)$	Heavy side pressure.
8. Squeezing rock, great depth	(2.10 to 4.50) $(B+H_t)$	
9. Swelling rock	Up to 250 ft. irrespective of value of $(B+H_t)$	Very heavy pressures.

Notes:

1. Above values apply to tunnels at depth greater than 1.5 $(B+H_t)$.
2. The roof of the tunnel is assumed to be located below the water table. If it is located permanently above the water table, the values given for rock conditions 4 to 6 can be reduced by fifty percent.
3. Some very dense clays which have not yet acquired properties of shale rock may behave as squeezing or swelling rock.
4. Where sandstone or limestone contain horizontal layers of immature shale, roof pressures will correspond to rock condition "very blocky and seamy."

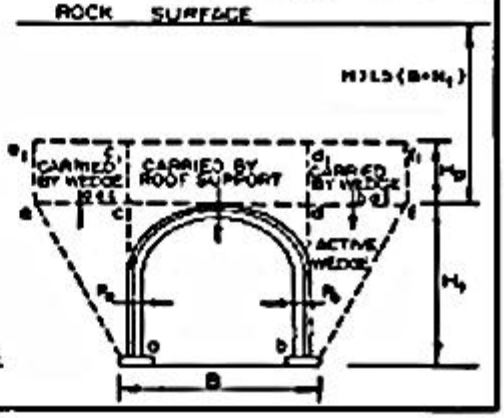


Table 1

Overburden rock load carried by roof support

5.4 OPENINGS IN SOFT GROUND.

5.4.1 GROUND BEHAVIOR. The method of construction of tunnels depends upon the response of the ground during and after excavations. The stand up time depends upon the type of soil, the position of groundwater, and the size of opening. Depending upon the response during its movement period, the ground is classified as: (1) firm, (2) raveling, (3) running, (4) flowing, (5) squeezing or (6) swelling.

5.4.1.1 IN FIRM GROUND, no roof support is needed during excavation and there is no perceptible movement.

5.4.1.2 IN RAVELING GROUND, chunks or flakes of material begin to fall prior to installing the final ground supports. Stand up time decreases with increasing size of excavation. With rising groundwater, raveling ground may become running ground. Sand with clay binder is one example of this type of soil.

5.4.1.3 IN RUNNING GROUND, stand up time is zero. The roof support must be inserted prior to excavation. Removal of side supports results in inflow of material which comes to rest at its angle of repose. Dry cohesionless soils fall into this category.

5.4.1.4 FLOWING GROUND acts as a thick liquid and it invades the opening from all directions including the bottom. If support is not provided, flow continues until the tunnel is completely filled. Cohesionless soil below groundwater constitutes flowing ground.

5.4.1.5 SQUEEZING GROUND advances gradually into the opening without any signs of rupture. For slow advancing soil, stand up time is adequate, yet the loss of ground results in settlement of the ground surface. Soft clay is a typical example of squeezing ground.

5.4.1.6 SWELLING GROUND advances into the opening and is caused by an increase in volume due to stress release and/or moisture increase. Pressures on support members may increase substantially even after the movement is restrained.

5.4.2 LOSS OF GROUND. As the underground excavation is made, the surrounding ground starts to move toward the opening. Displacements result from stress release, soil coming into the tunnel from raveling, runs, flows, etc. The resulting loss of ground causes settlement of the ground surface. The loss of ground associated with stress reduction can be predicted reasonably well, but the ground loss due to raveling, flows, runs, etc. requires a detailed knowledge of the subsurface conditions to avoid unacceptable amounts of settlement.

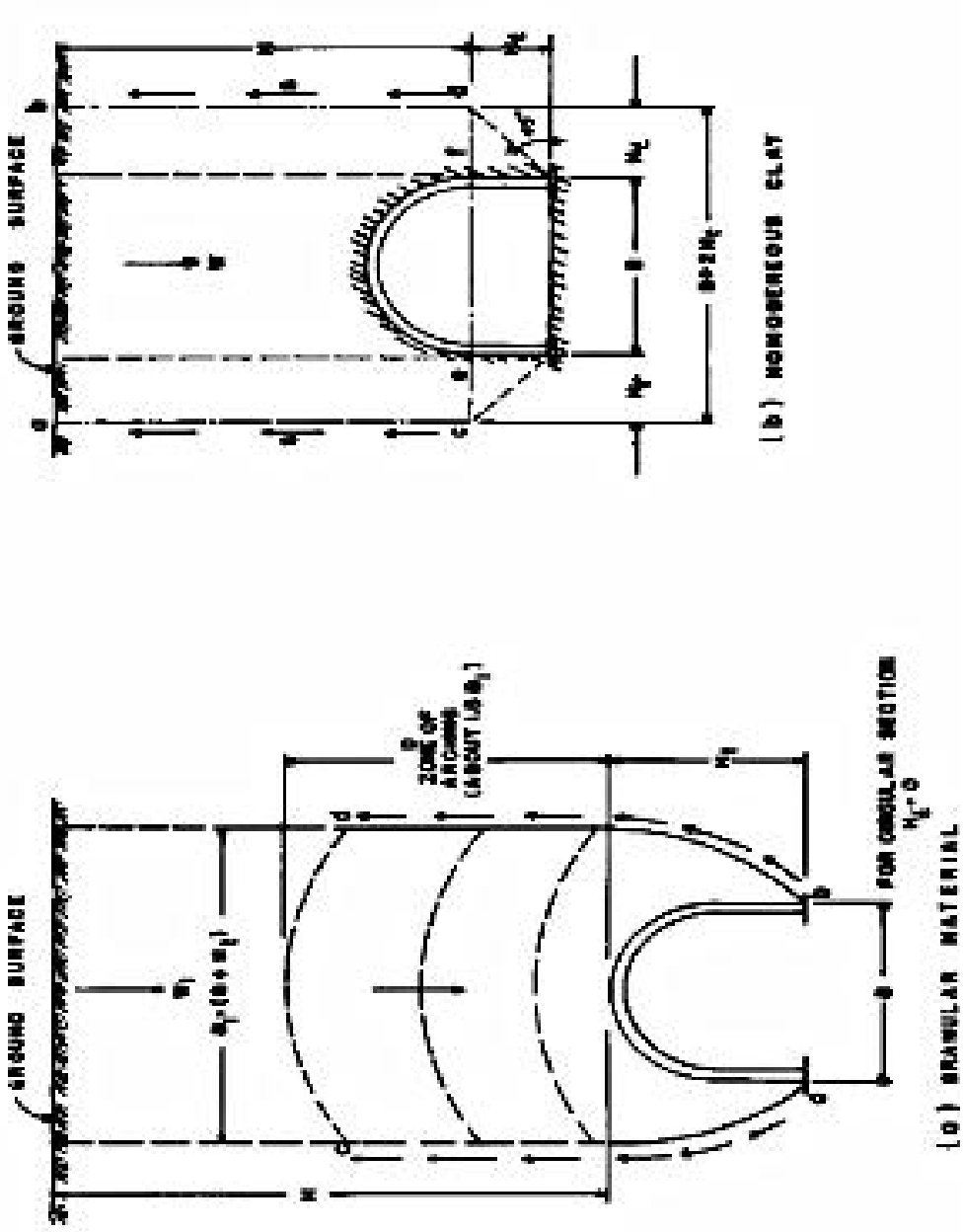


Figure 23

Load actions on underground openings in earth

5.4.3 LOADS. The support pressures in the underground openings are governed by the unit weight of the soil, groundwater table, soil properties, deformations during excavation, interaction between soil and the supports, shape of the opening, and the length of time that has elapsed since the installation of lining. Other factors such as the presence of another opening adjacent to it, excavation of a large deep basement near an existing opening, load from neighboring structures, and change in groundwater conditions, will also affect the design pressures on the tunnel supports. A schematic representation of the load action on underground openings is shown in Figure 23. Estimate of load for temporary supports in earth tunnels may be obtained from Table 2.

5.5 PRESSURE ON VERTICAL SHAFTS.

5.5.1 SHAFT IN SAND. In the excavation of a vertical cylindrical shaft granular soils, pressures surrounding the shaft approach active values. If outward directed forces from a buried silo move the silo walls into the surrounding soil, pressures approach passive values as an upper limit.

5.5.1.1 PRESSURE COEFFICIENTS. See Figure 24 for active and passive pressure coefficients for a cylindrical shaft of unlimited depth in granular soils.

5.5.1.2 MODIFICATION OF ACTIVE PRESSURES. For relatively shallow shafts (depth less than twice the diameter), rigid bracing at the top may prevent development of active conditions. In this case, horizontal pressures may be as large as at-rest pressures on a long wall with plane strain in the surrounding soil.

5.5.1.3 IF GROUNDWATER IS ENCOUNTERED, use submerged unit weight of sand and add hydrostatic pressure.

5.5.2 SHAFT IN CLAY.

5.5.2.1 PRESSURE ON WALLS OF SHAFTS IN SOFT CLAY. For a cylindrical shaft, no support is needed from the ground surface to a depth of

$$Z_o = 2C/\upsilon$$

To determine the approximate value of ultimate horizontal earth pressure on a shaft lining at any depth z, use

$$ph = \upsilon \times (z - c)$$

where

υ = effective unit weight of clay
z = depth
c = cohesion

This pressure is likely to occur after several months.

Type of ground	Ground condition	Design load, H_p	Remarks
Running ground above water table	Loose	0.50 (B + H_t)	
	Medium	0.40 (B + H_t)	
	Dense	0.30 (B + H_t)	
Running ground in compressed air tunnel		Disregard air pressure; H_p equal to that for running ground, above water table with equal density	
Flowing ground in free air tunnel		H or 2 (B + H_t), whichever is smaller	
Raveling ground	Above water table	[(T – t)/T] x H_p (running)	
	Below water table (free air)	[(T – t)/T] x H_p (running)	
	Below water table (compressed air)	[(T – t)/T] x (2H_p - υ)	
Squeezing ground	Homogeneouse	H – (P_c/υ) – Hq_u/2υ(B + 2H_t)	After complete blowout
	Soft roof, stiff sides	H – (P_c/υ) – Hq_u/2υ(B)	P_c = 0
	Stiff roof, soft sides	H – (P_c/υ) – Hq_u/2υ(B + 6H_t)	
Swelling ground	Intact	Very small	Permanent roof support should be completed within a few days after mining
	Fissured, above water table	H_p, equal to that for raveling ground with same stand up time H	
	Fissured, below water table, free air tunnel		

p_c = air pressure in pounds per square foot
q_u = unconfined compressive strength of ground above roof in pounds per square foot
υ = unit weight of soil in pounds per cubic foot
t = stand up time, minutes
T = elapsed time between excavating and completion of permanent structure, minutes
H = vertical distance between ground surface and tunnel roof in feet
H_p = design load in feet of earth, see table 1
H_t = height of tunnel , see table 1
B = width of tunnel, see table 1
[*] for circular tunnels H_t = 0, B = diameter

Table 2

Loads for temporary supports in earth tunnels

At depths more than 1.5 (B + H_t)

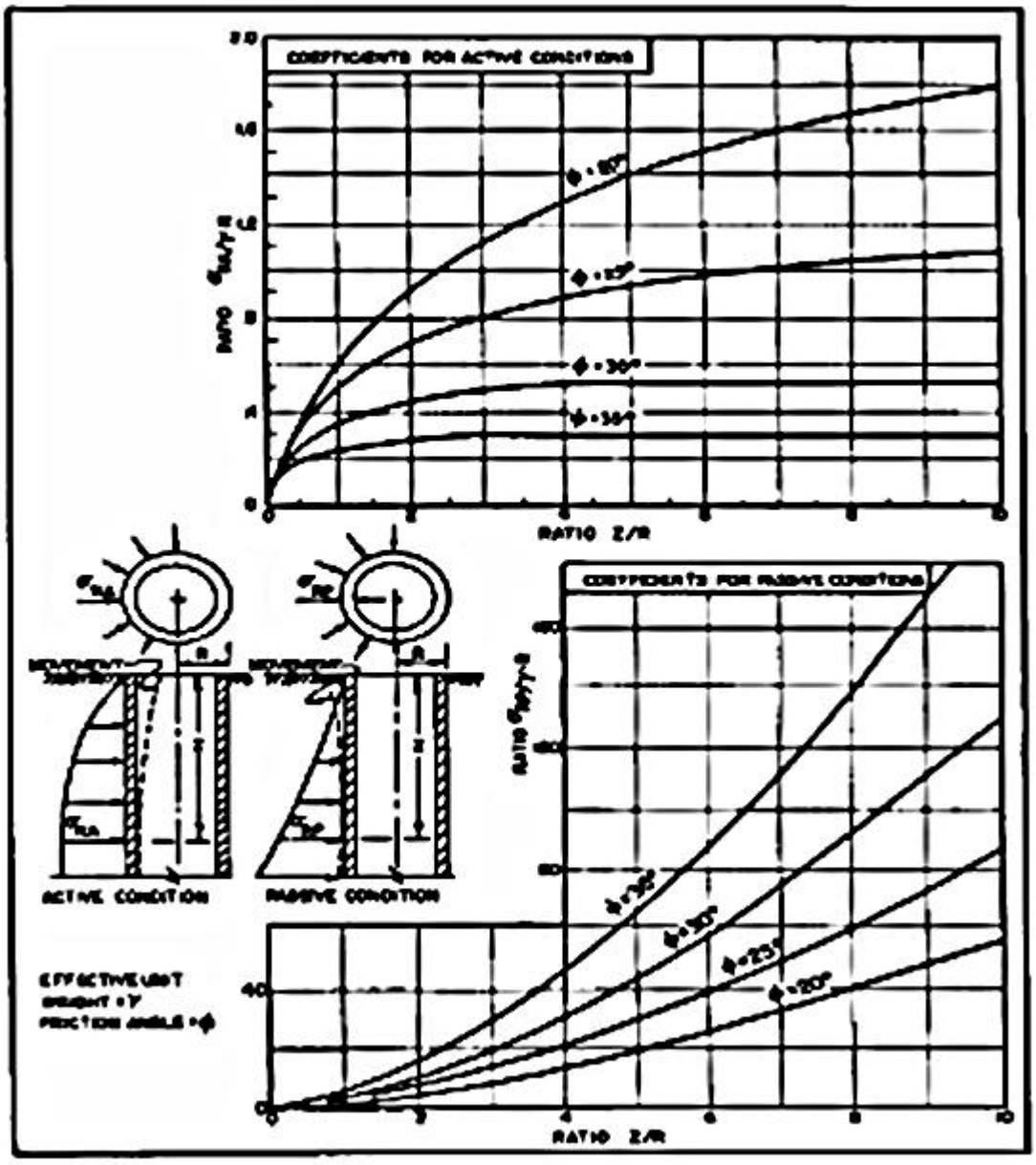


Figure 24

Coefficient for active or passive pressures

on underground cylindrical shafts or silos

5.1.2.2 Pressure on Walls of Shafts in Stiff Clay. On shafts located in stiff, intact, or fissured swelling clays, initially the pressure on the shaft lining is very small. Over a period of time, the pressure may increase to several times the overburden pressure (i.e., ultimately to the swelling pressure if shaft lining is sufficiently rigid). Local experience in that soil or field measurements can provide useful information. For further details of pressures on shafts, see Reference 23.

6. NUMERICAL STRESS ANALYSIS

Stress analysis using numerical methods and computers are available for many simple as well as more complex loading conditions.

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QUIZ

An Introduction to Distribution of Stresses in Soil

1. Stresses in earth masses are analyzed using two basic and different assumptions. One assumes elastic conditions, and the other assumes full mobilization of shear strength (plastic equilibrium). Elastic solutions apply to problems for which shear failure is unlikely. If the safety factor against shear failure exceeds about _____, stresses are roughly equal to values computed from elastic theory.

a. 6
b. 5
c. 4
d. 3

2. If normal and shear stresses at one orientation on an element in an earth mass are known, stresses at all other orientations may be determined from _____ .

a. Bernoulli's equation
b. Mohr's circle
c. the Hazen-Williams chart
d. the Chezy-Manning equation

3. Division of weight of overlying soil and water into effective stress and pore water pressure depends on the position of the groundwater table or the flow field induced by seepage. For static water condition, effective stresses at any point below the groundwater level may be computed using the total unit weight of soil above the water level and _____ below the water level.

a. extrapolation
b. buoyant unit weight
c. linear analysis
d. interpolation

4. Division of applied load between _____ and effective stress is a function of the boundary conditions, the stress-strain properties, and the permeability of the stressed and surrounding soils.

a. structural loading
b. overburden
c. actual strain
d. pore pressure

5. Actual foundation conditions differ from the homogeneous isotropic, semi-infinite mass assumed in the _____ expressions. The modulus of elasticity usually varies from layer to layer, and soil deposits frequently are more rigid in the horizontal direction than in the vertical.

a. time of consolidation
b. Westergaard
c. Boussinesq
d. anisotropic

6. The Westergaard analysis is based on the assumption that the soil on which load is applied is reinforced by closely spaced horizontal layers which _____.

a. consolidate at a uniform rate
b. are of uniform depth
c. prevent horizontal displacement
d. are cohesionless

7. When the foundation soil consists of a number of layers of _____, having distinctly different elastic properties, the vertical and other stresses are markedly different from those obtained by using the Boussinesq equation.

a) cohesive soil
b) cohesionless soil
c) substantial thickness
d) sand

8. If a rigid surface layer over a weaker underlying layer is substantially more rigid, it acts as a distributing mat and the _____ in the underlying soil layer are less than Boussinesq values.

a. shear stresses
b. vertical stresses
c. horizontal shear zones
d. consolidation times

9. If there is no distinct change in the character of subsurface strata within the critical depth, elastic solutions for layered foundations need not be considered. Critical depth is the depth below the foundation within which soil compression contributes significantly to surface settlements. For fine-grained compressible soils, the critical depth extends to that point where applied stress decreases to 10 percent of effective overburden pressure. In coarse-grained material critical depth extends to that point where applied stress decreases to _____ percent of effective overburden pressure.

a. 20
b. 30
c. 40
d. 50

10. Pressures acting on shallow buried pipe and conduits are influenced by the relative rigidity of the _____ and surrounding soil, depth of cover, type of loading, span (maximum width) of structure, method of construction, and shape of pipe.

a. rock
b. structural load
c. overburden
d. pipe

11. To design a _____, the computed loads (dead and live) are modified to account for bedding conditions and to relate maximum allowable load to the three-edge bearing test load D. See ASTM C76, Reinforced Concrete Culvert, Storm Drain, and Sewer Pipe, for test standards for D load.

a. semi-rigid pipeline
b. rigid conduit
c. corrugated aluminum conduit
d. corrugated steel conduit

12. Corrugated or thin wall smooth steel pipes are sufficiently _____ to develop horizontal restraining pressures approximately equal to vertical pressures if backfill is well compacted.

a. strong
b. plastic
c. rigid
d. flexible

13. The method of construction of _____ depends upon the response of the ground during and after excavations. The stand-up time depends upon the type of soil, the position of groundwater, and the size of opening. Depending upon the response during its movement period, the ground is classified as: (1) firm, (2) raveling, (3) running, (4) flowing, (5) squeezing or (6) swelling.

a. tunnels
b. foundations
c. caissons
d. large diameter pipelines

14. In _____ ground, chunks or flakes of material begin to fall prior to installing the final ground supports.

a. sluffing
b. raveling
c. cohesionless rock
d. cohesive sand

15. In running ground, stand up time is _____ .

a. indeterminate
b. high
c. low
d. zero